



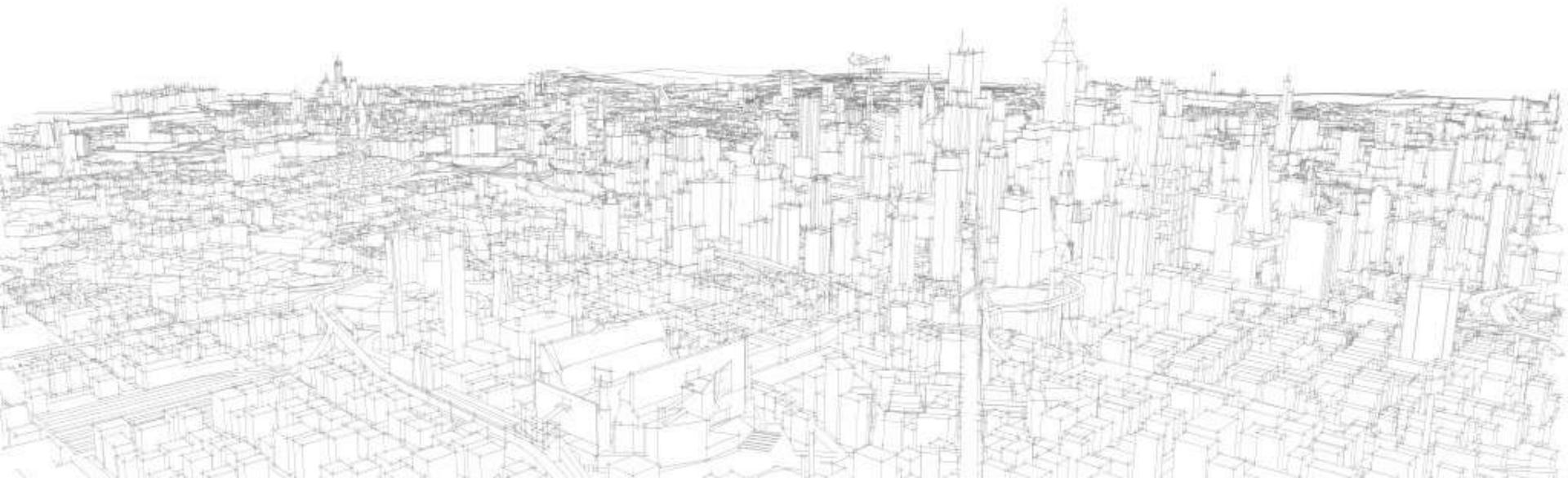
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# Urban morphology and structural invariants in street networks

Alec Kirkley<sup>1</sup>, Hugo Barbosa<sup>1</sup>, Marc Barthelemy<sup>2</sup> and Gourab Ghoshal<sup>1</sup>

<sup>1</sup>Department of Physics & Astronomy, University of Rochester

<sup>2</sup>Institut de Physique Theorique, CEA, Gif-sur-Yvette, France











# Long-term real growth in US Stocks (Log Scale)

Annual price index adjusted for inflation 1871–2010

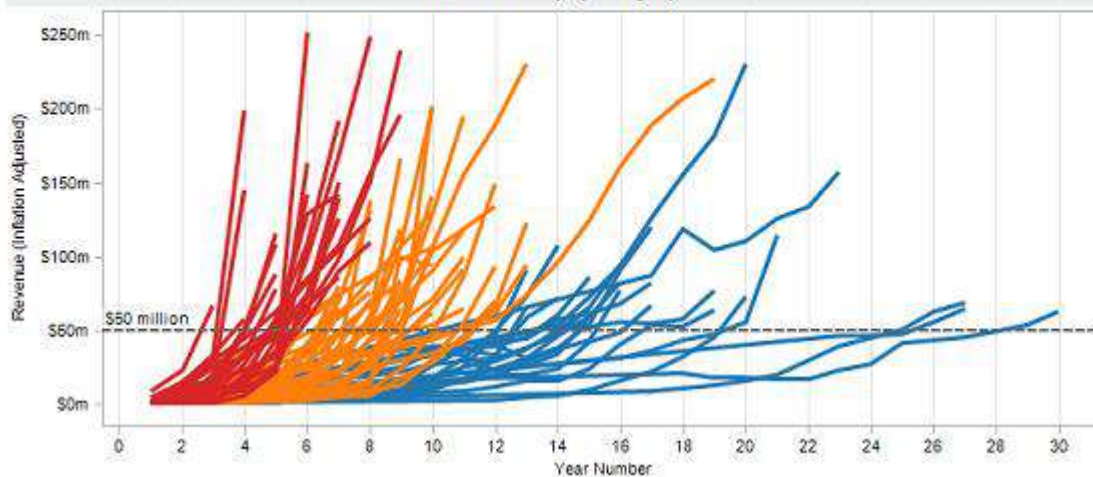


## Tale of 100 Entrepreneurs

Click to interact

Rocket Ship Hot Company Slow Burner

### Growth History by Company



# Growth, innovation, scaling, and the pace of life in cities

Luis M. A. Bettencourt<sup>\*†</sup>, José Lobo<sup>‡</sup>, Dirk Helbing<sup>§</sup>, Christian Kühnert<sup>§</sup>, and Geoffrey B. West<sup>\*¶</sup>

Table 1. Scaling exponents for urban indicators vs. city size

Y	$\beta$	95% CI	Adj- $R^2$
New patents	1.27	[1.25,1.29]	0.72
Inventors	1.25	[1.22,1.27]	0.76
Private R&D employment	1.34	[1.29,1.39]	0.92
"Supercreative" employment	1.15	[1.11,1.18]	0.89
R&D establishments	1.19	[1.14,1.22]	0.77
R&D employment	1.26	[1.18,1.43]	0.93
Total wages	1.12	[1.09,1.13]	0.96
Total bank deposits	1.08	[1.03,1.11]	0.91
GDP	1.15	[1.06,1.23]	0.96
GDP	1.26	[1.09,1.46]	0.64
GDP	1.13	[1.03,1.23]	0.94
Total electrical consumption	1.07	[1.03,1.11]	0.88
New AIDS cases	1.23	[1.18,1.29]	0.76
Serious crimes	1.16	[1.11, 1.18]	0.89

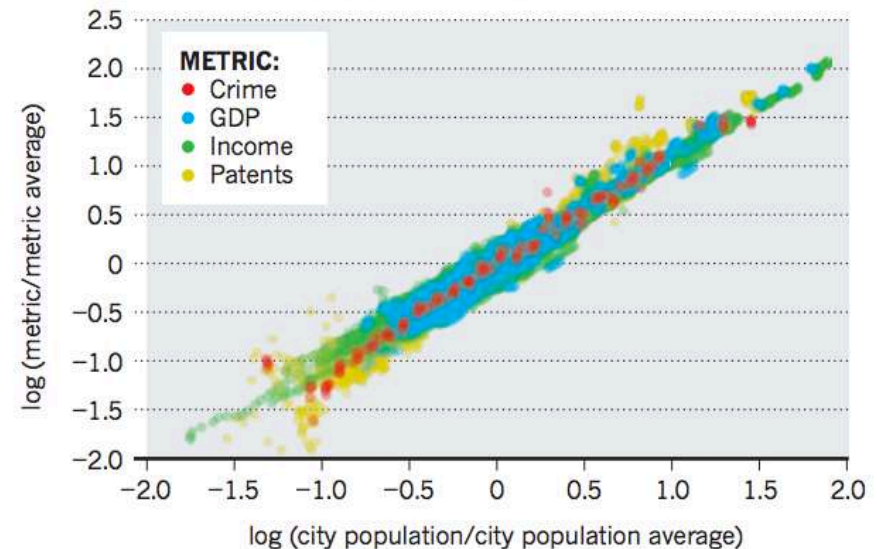
$$1.1 \leq \beta \leq 1.3$$

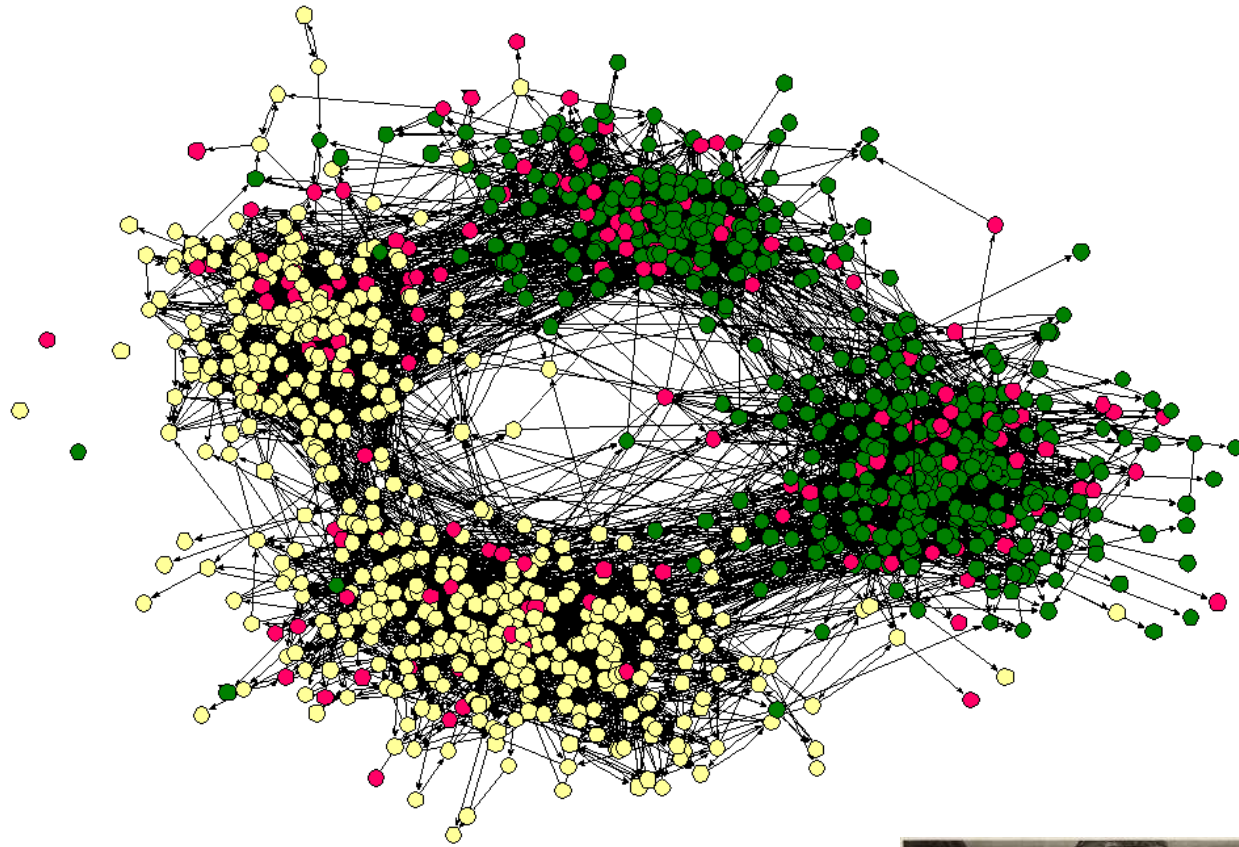
Urban indicators scale **super-linearly** with population size

$$Y(t) \sim N(t)^\beta$$

## PREDICTABLE CITIES

Data from 360 US metropolitan areas show that metrics such as wages and crime scale in the same way with population size.





What is a city but the people?

-William Shakespeare, Tragedy of Coriolanus





ARTICLE

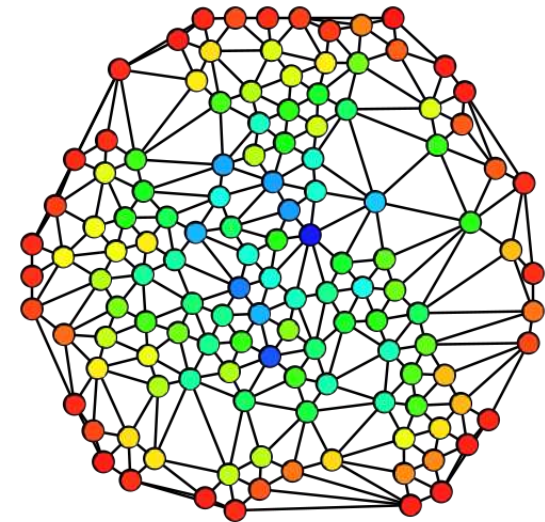
Received 4 Dec 2012 | Accepted 30 Apr 2013 | Published 4 Jun 2013

DOI: 10.1038/ncomms2961

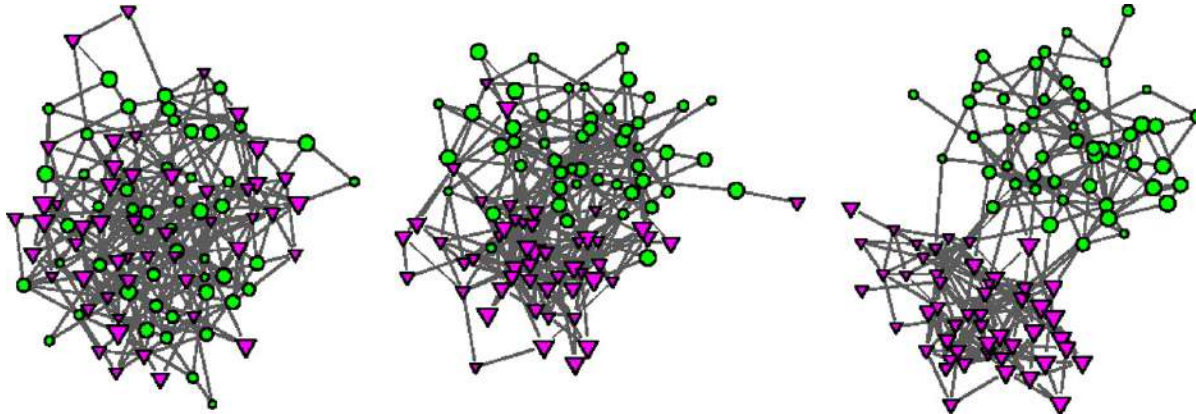
# Urban characteristics attributable to density-driven tie formation

Wei Pan<sup>1</sup>, Gourab Ghoshal<sup>1,†</sup>, Coco Krumme<sup>1</sup>, Manuel Cebrian<sup>1,2,3</sup> & Alex Pentland<sup>1</sup>

- **Generative theory** that **links** urban **geography** and **population density** and naturally reproduces scaling behavior found in urban data.
- The density of **social ties** is a scaffold on which rate of information exchange and interactions takes place.
- Thus this may well be **one** of the primary mechanisms behind innovation and productivity.



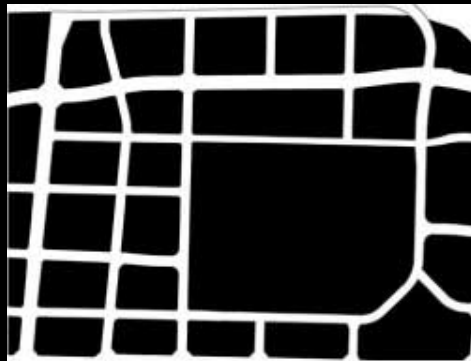
# Limitations



- While the network is growing, it is static in the sense that there is no opportunity to reshape links.
- Relations are treated homogeneously, in the sense that no differentiation based on social, functional, commuting behavior.
- Most importantly the predictions of the model fail when compared to economic data from developing countries.







**MISSISSAUGA**



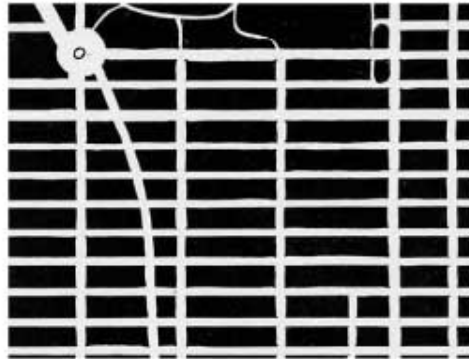
**BARCELONA**



**COPENHAGEN**



**LONDON**



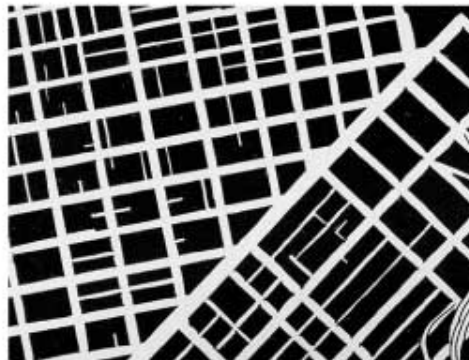
**NEW YORK**



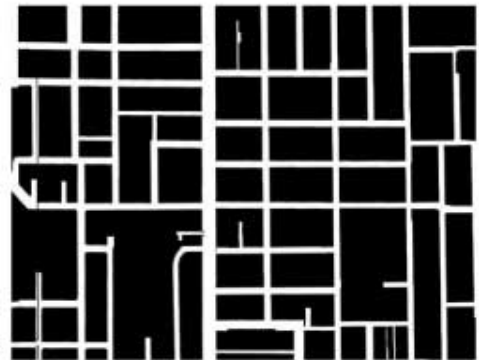
**PARIS**



**ROME**



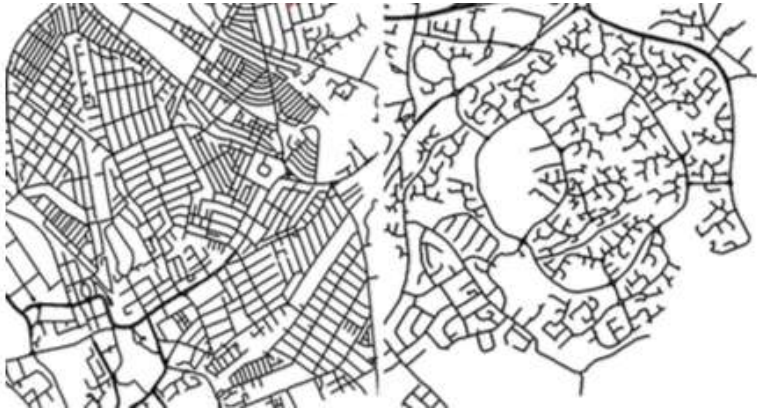
**SAN FRANCISCO**



**TORONTO**

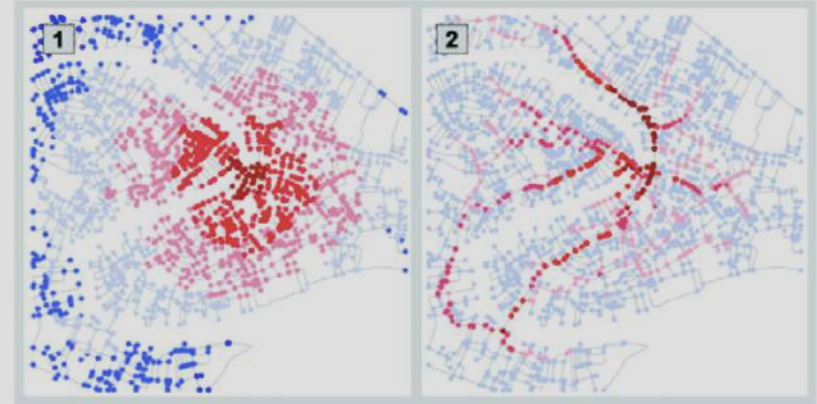
# Urban Street networks

## Fractal structure



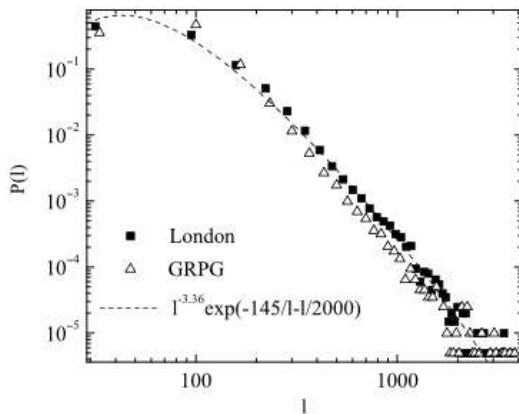
R. Murcio et al. (2015) PRE

## Street network analysis



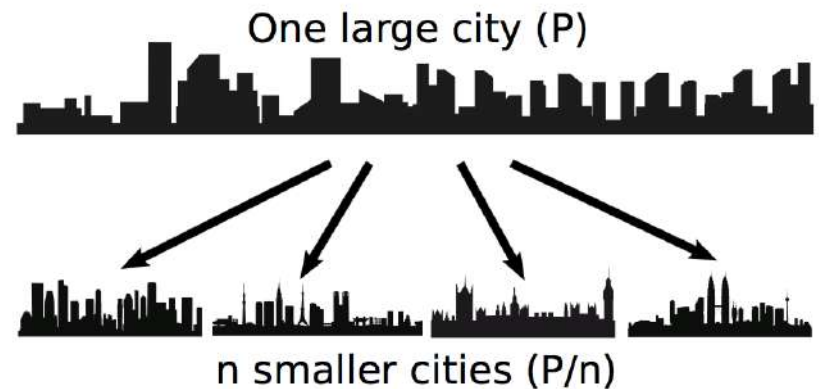
Crucitti et al. (2006) PRE

## Statistical properties



Masucci (2009) Eur. Phys. J. B

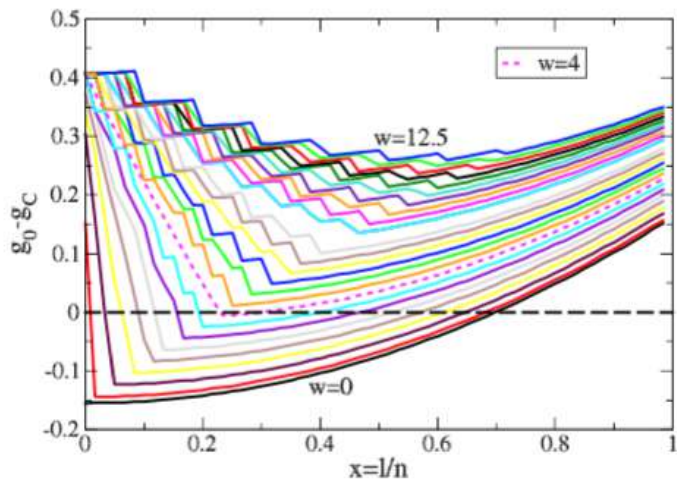
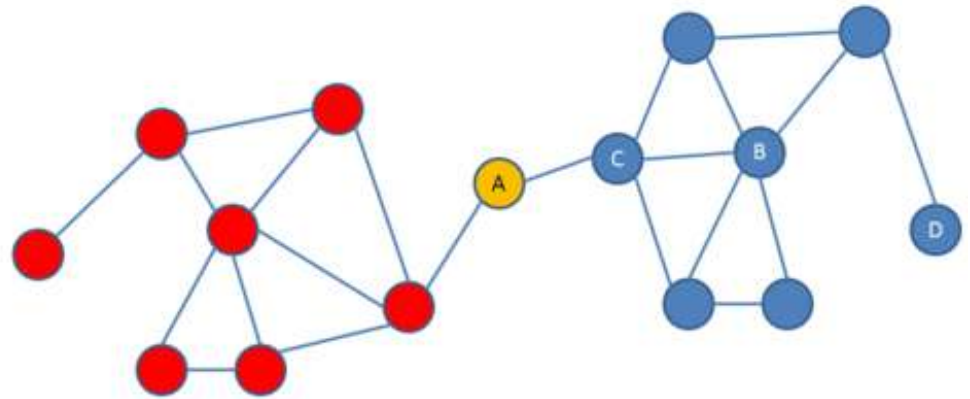
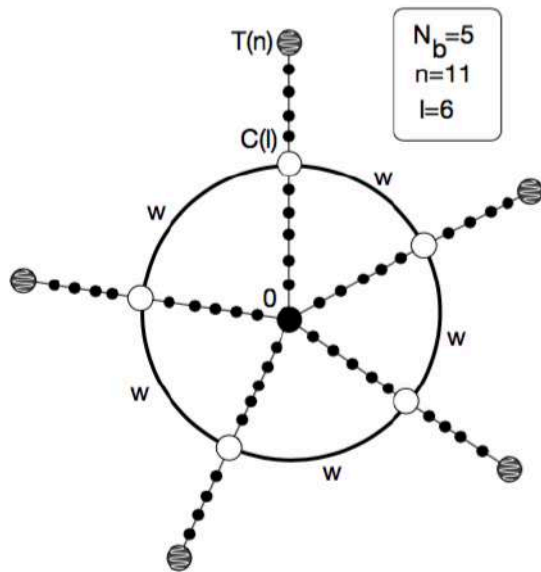
## Polycentric or monocentric



R. Louf and M. Barthelemy (2014) Scientific report



# Betweenness Centrality



$$g_B(v) = \mathcal{N} \sum_{s,t \in G} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

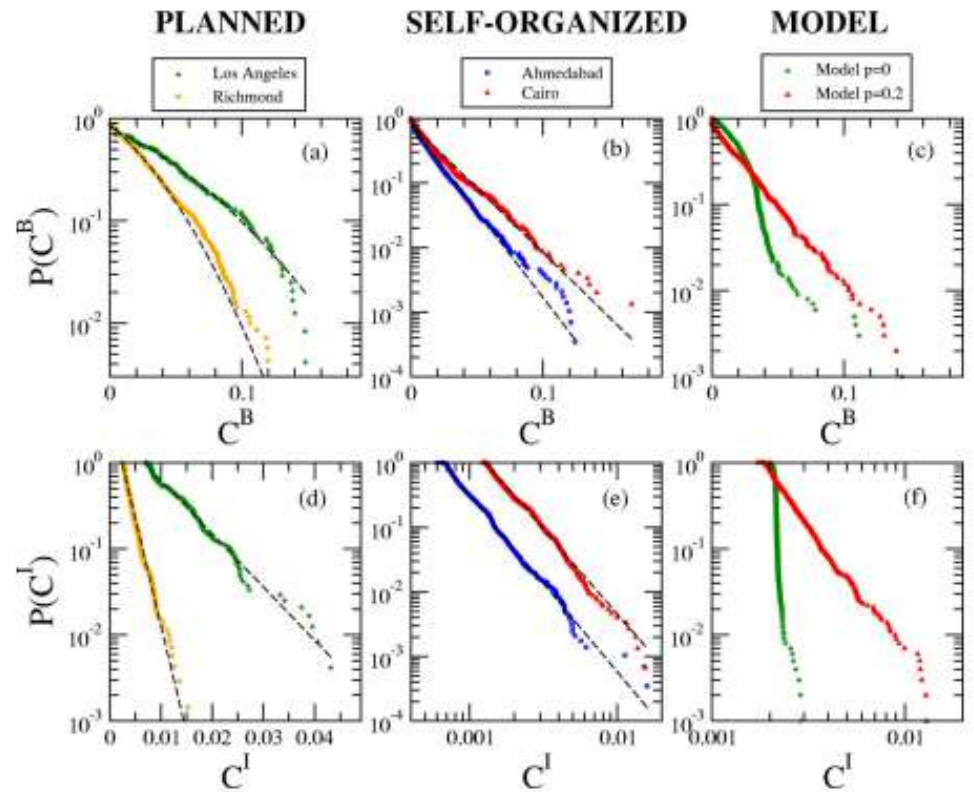
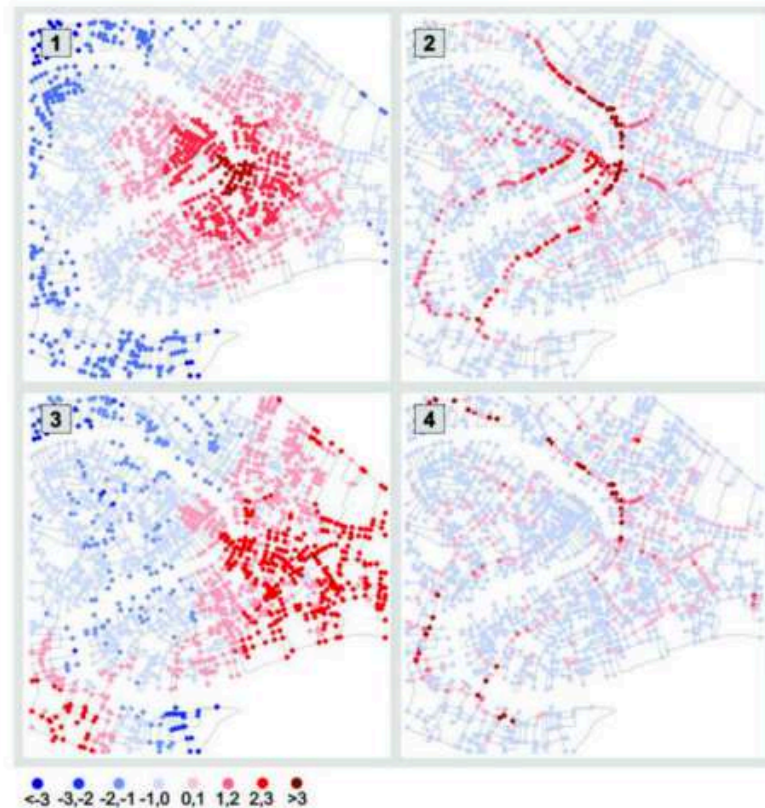
## Centrality measures in spatial networks of urban streets

Paolo Crucitti,<sup>1</sup> Vito Latora,<sup>2</sup> and Sergio Porta<sup>3</sup>  
<sup>1</sup>*Scuola Superiore di Catania, Italy*

<sup>2</sup>*Dipartimento di Fisica e Astronomia, Università di Catania, and INFN Sezione di Catania, Italy*

<sup>3</sup>*Dipartimento di Progettazione dell'Architettura, Politecnico di Milano, Italy*

(Received 20 October 2005; published 24 March 2006)







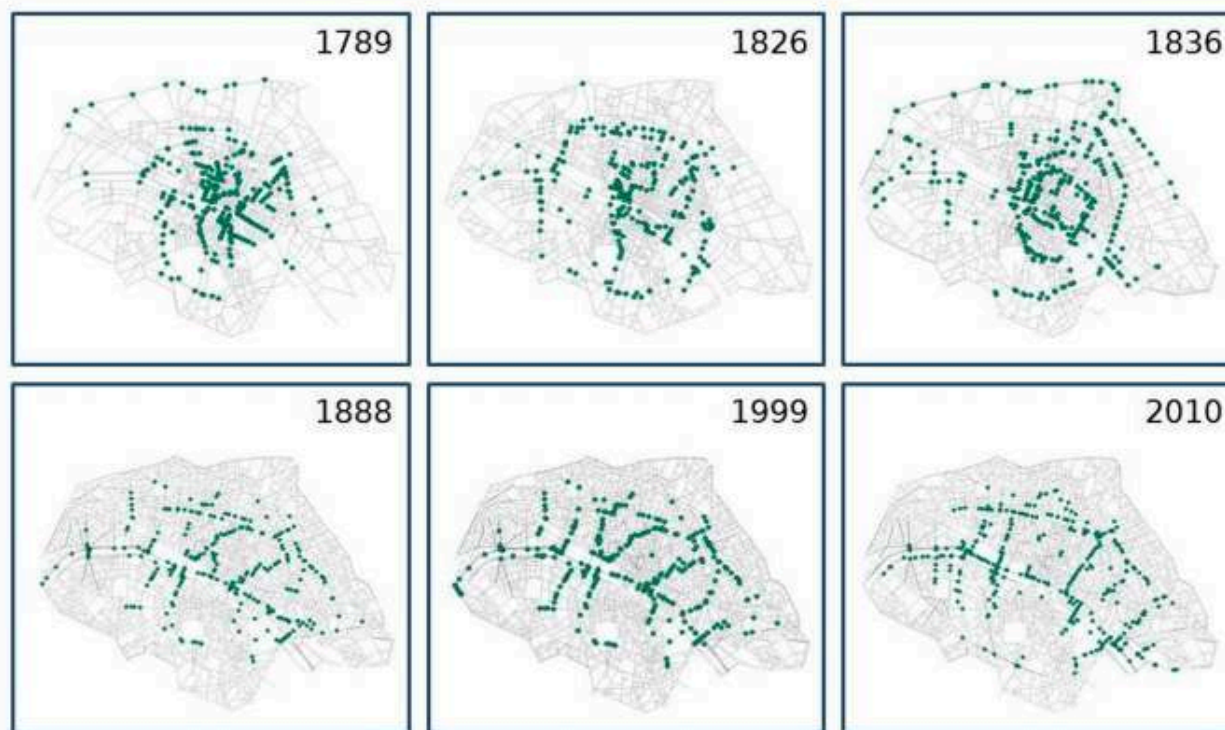
Altmetric: 31 Citations: 16

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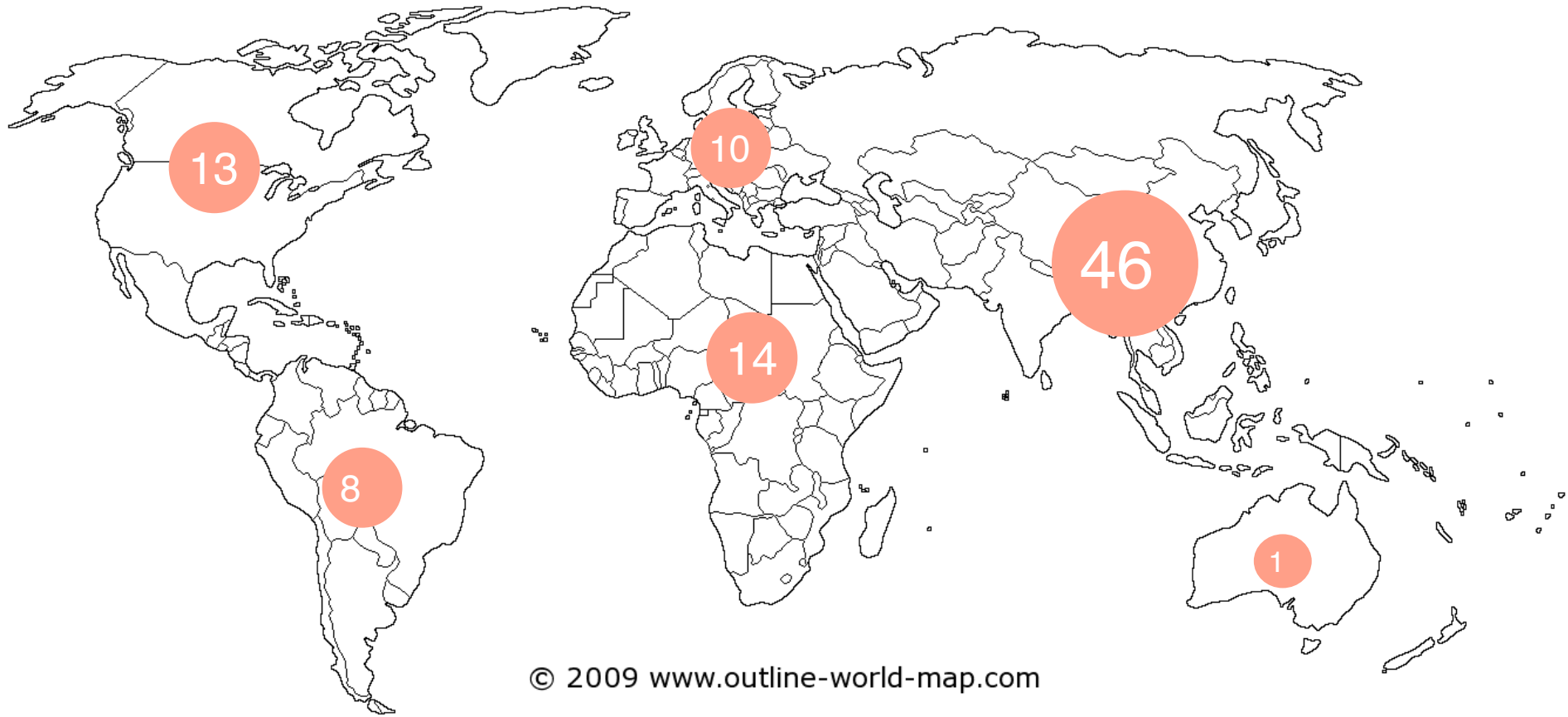
## Self-organization versus top-down planning in the evolution of a city

Marc Barthelemy , Patricia Bordin, Henri Berestycki & Maurizio Gribaudo



# Data Source

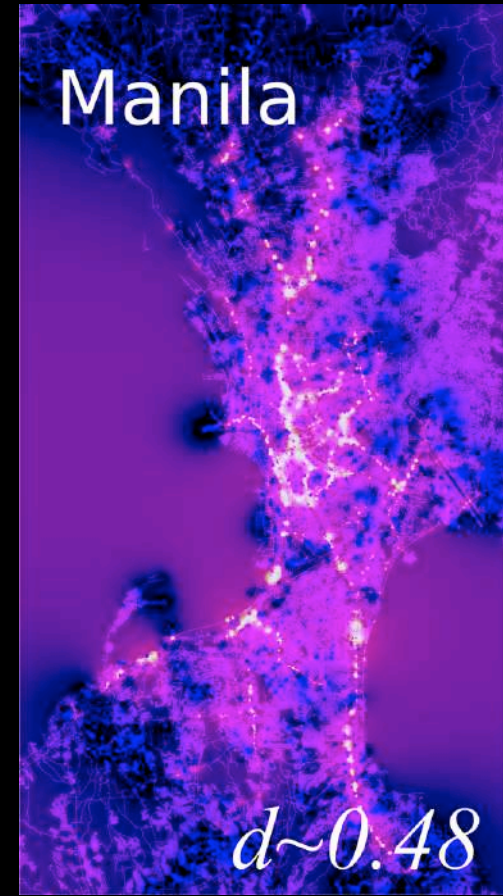
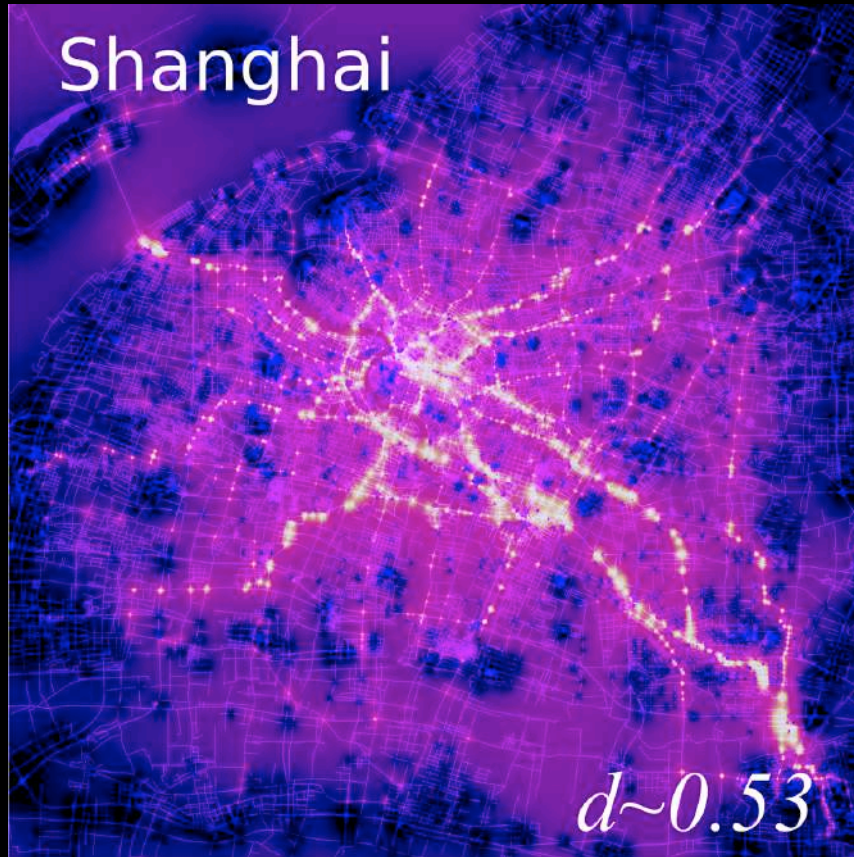
OpenStreetMap API  
92 most populous cities





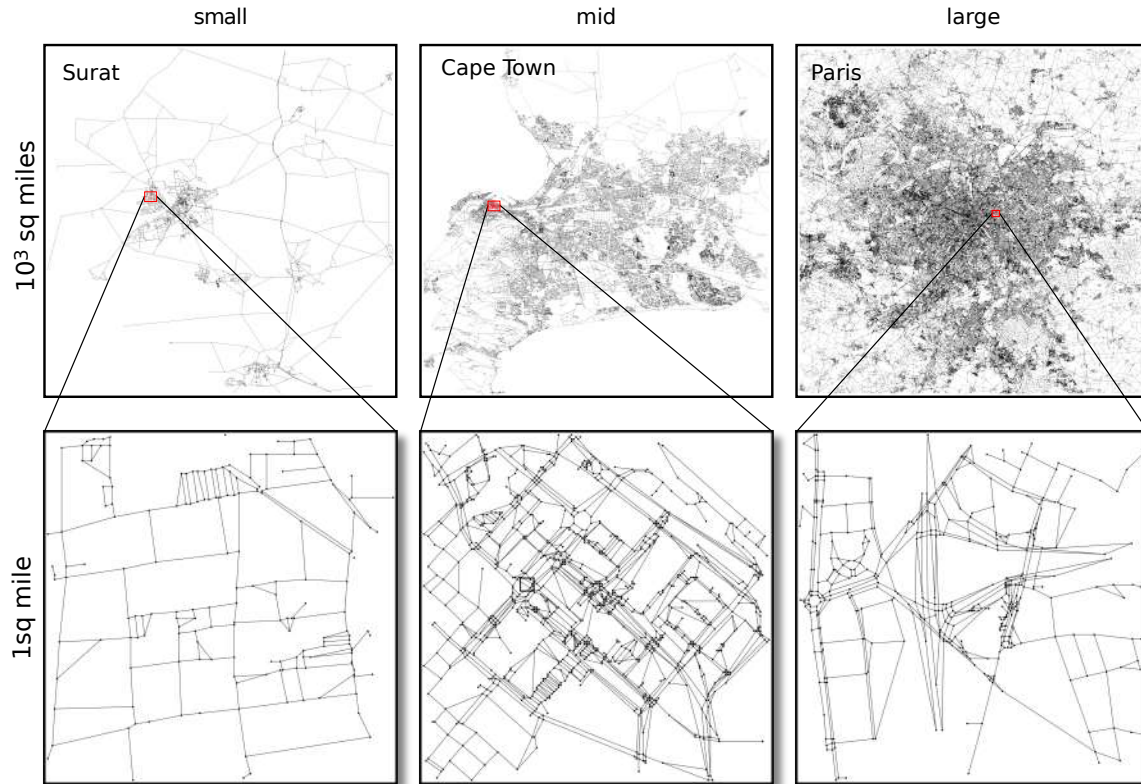


# Betweenness distribution of cities



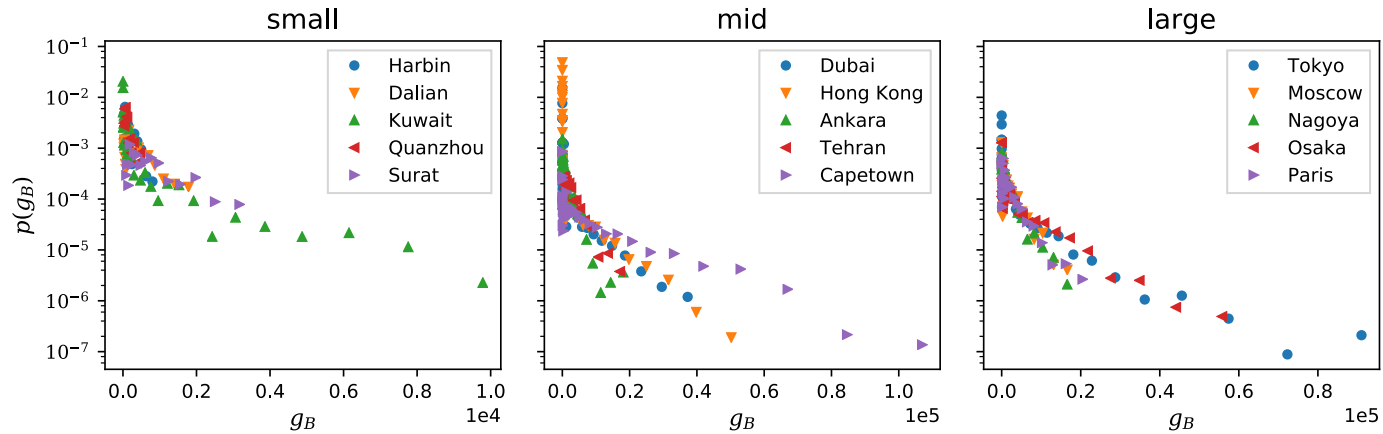


# Street network statistics

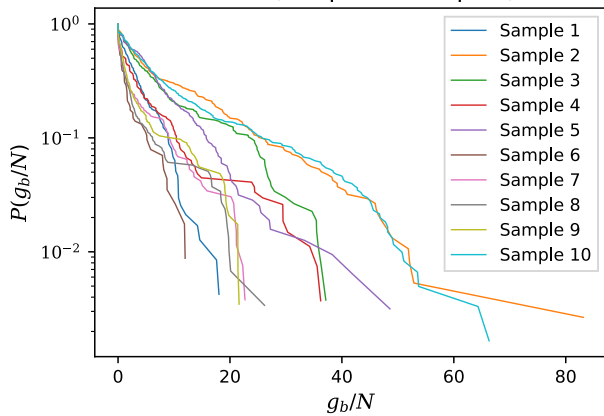


	Area $A$ (sq miles)	Nodes $N$	Length $\ell$ (miles)	Density $\rho$	Edges $e$
mean	1776	83529	10850	46	130253
stdev	744	90335	9353	40	143060
min	300	3349	1114	3	5020
25%	1229	18925	3597	14	28518
50%	1703	62451	7961	39	95797
75%	2268	118712	14758	69	178773
max	4464	612418	51316	242	976040

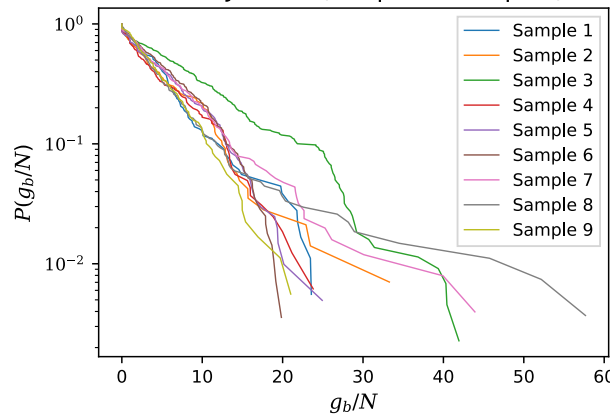
# Betweenness (1 sq. mile)



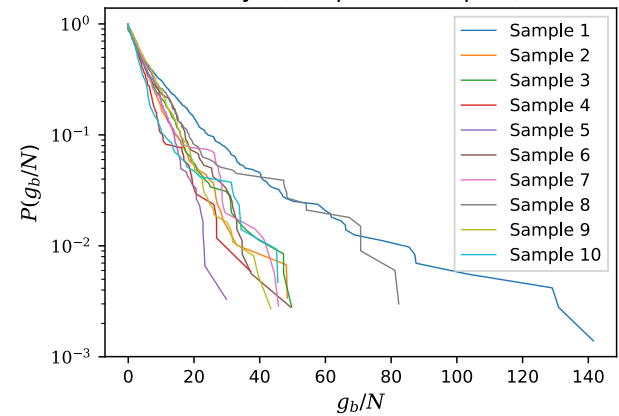
Manila (1-sq mile samples)



Rio de Janeiro (1-sq mile samples)

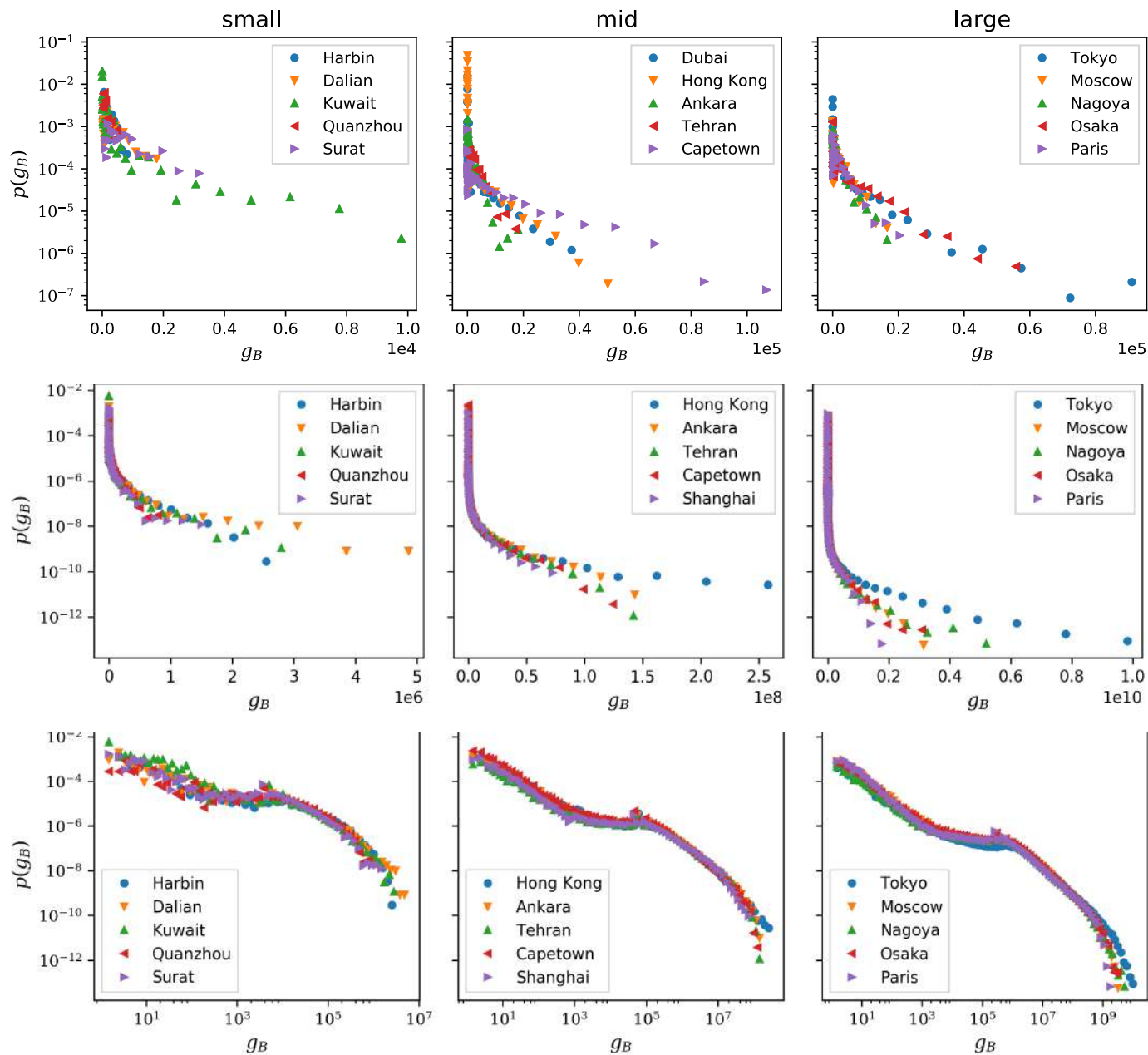


Tokyo (1-sq mile samples)

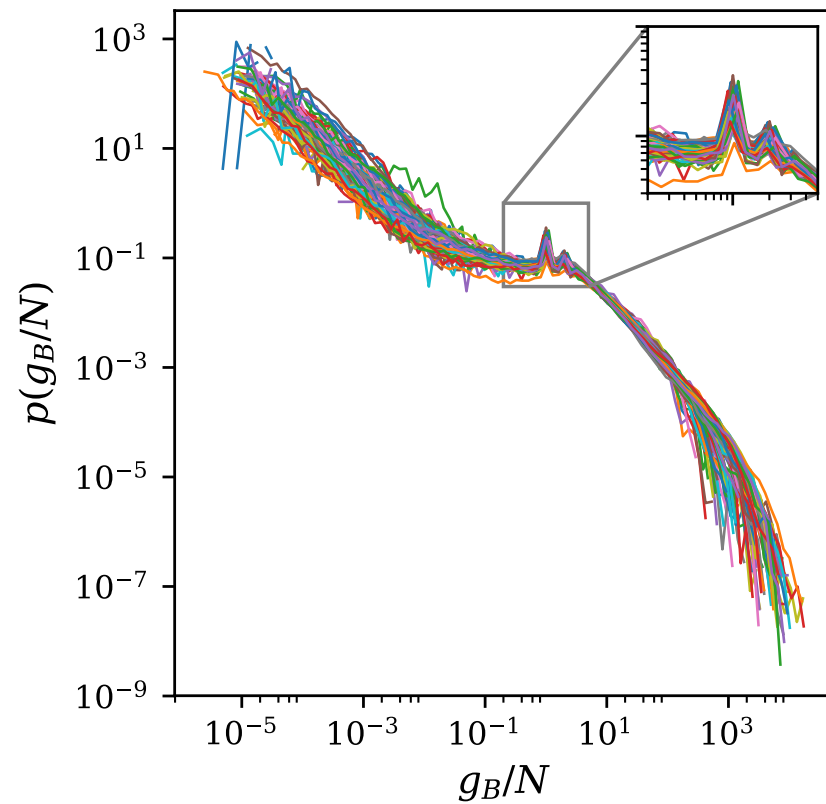
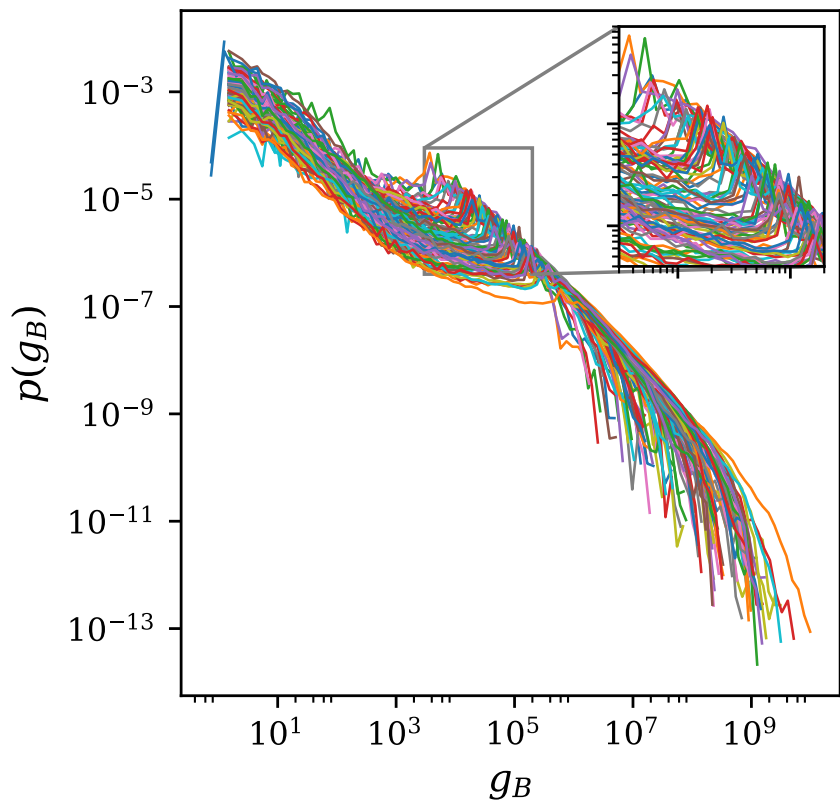




# Betweenness (different scales)



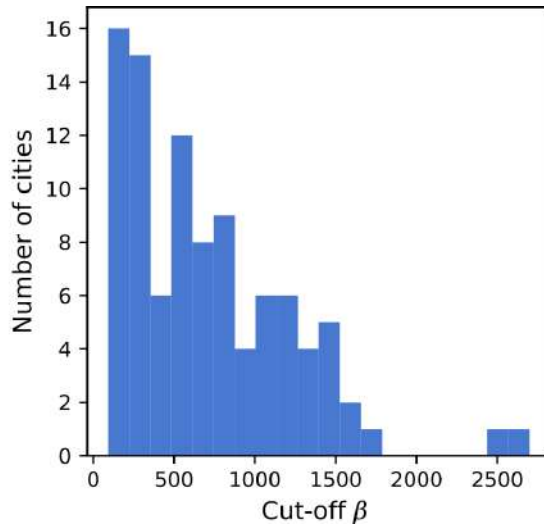
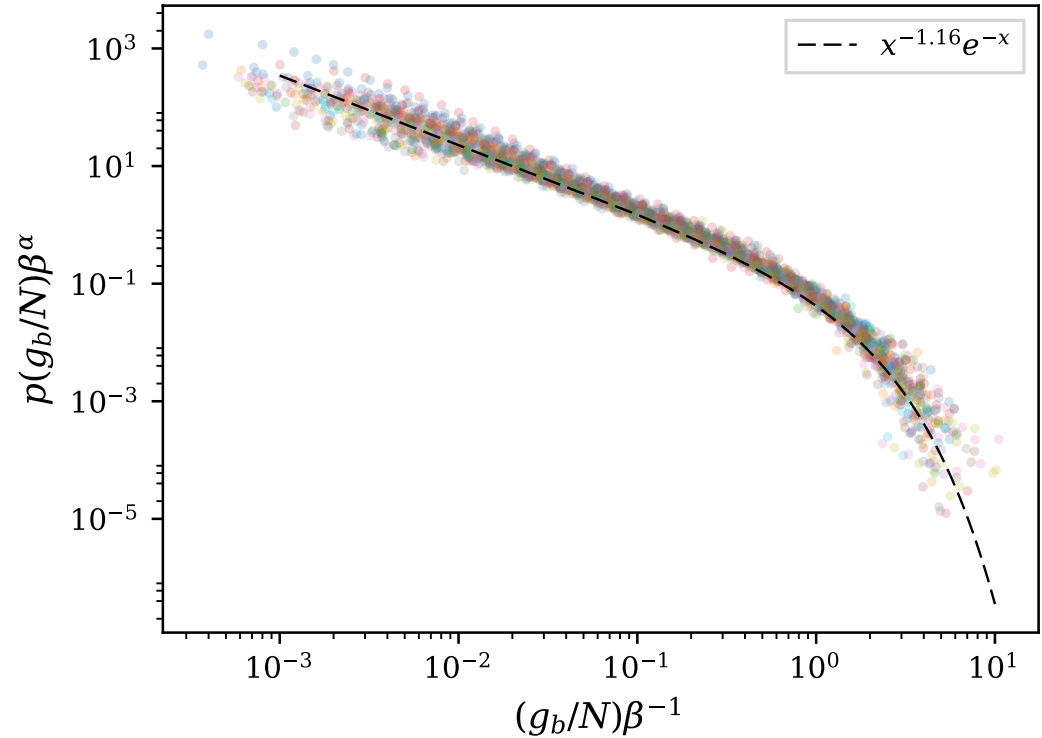
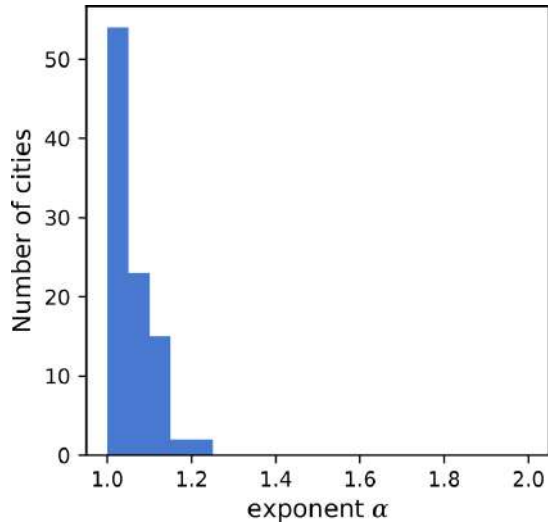
# Betweenness (1000 sq. miles)





# Betweenness invariance

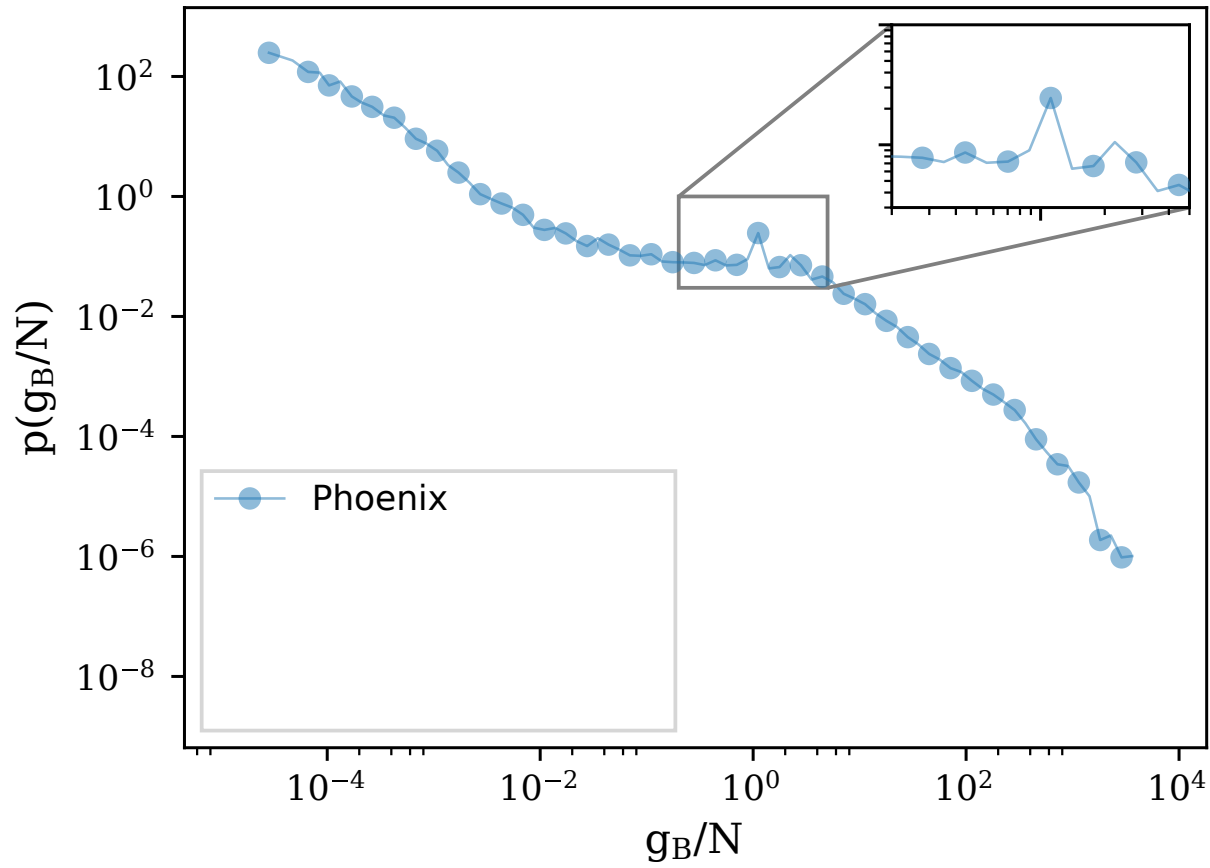
Truncated power law:  $p(\tilde{g}_B) \sim \tilde{g}_B^{-\alpha} e^{-\tilde{g}_B/\beta}$



After appropriate re-scaling,  
Betweenness distribution for all  
cities look the same!

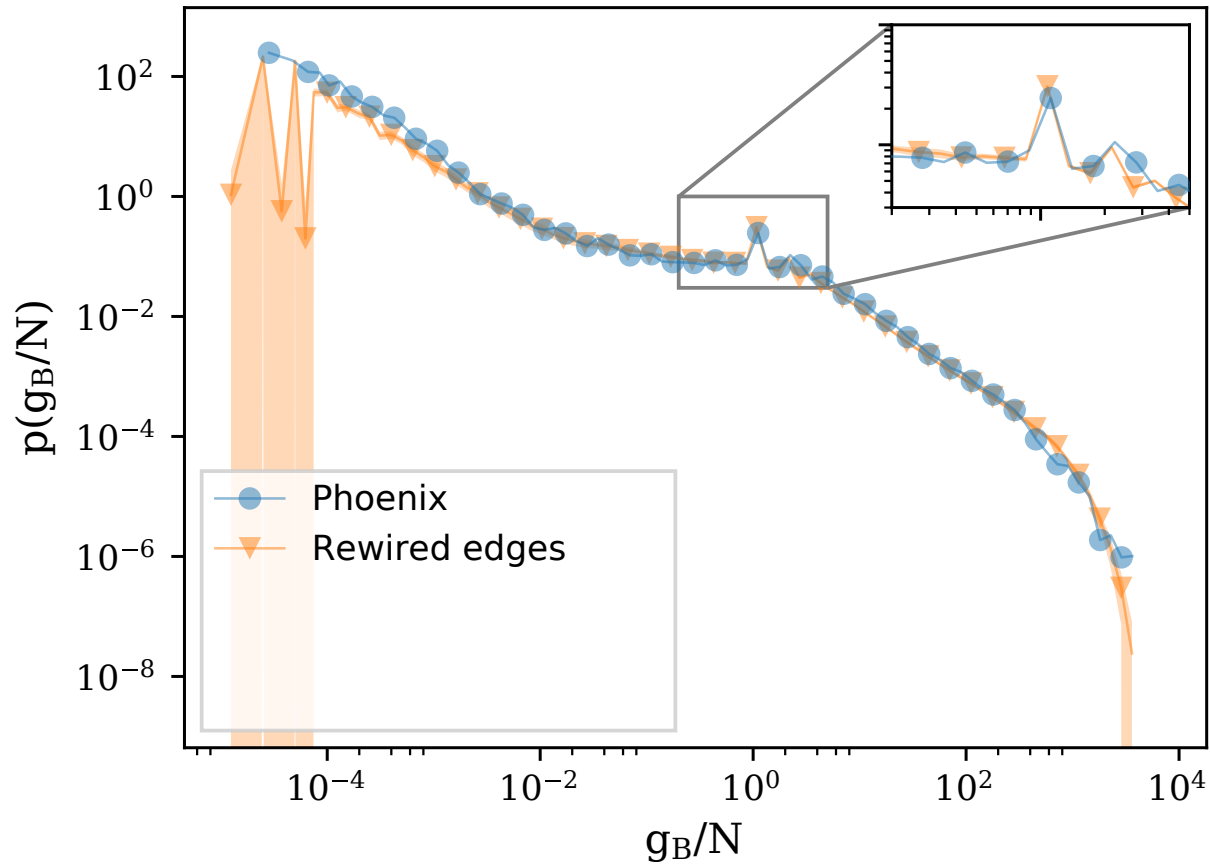
**What are the factors behind  
this remarkable invariance?**

# Random planar graphs

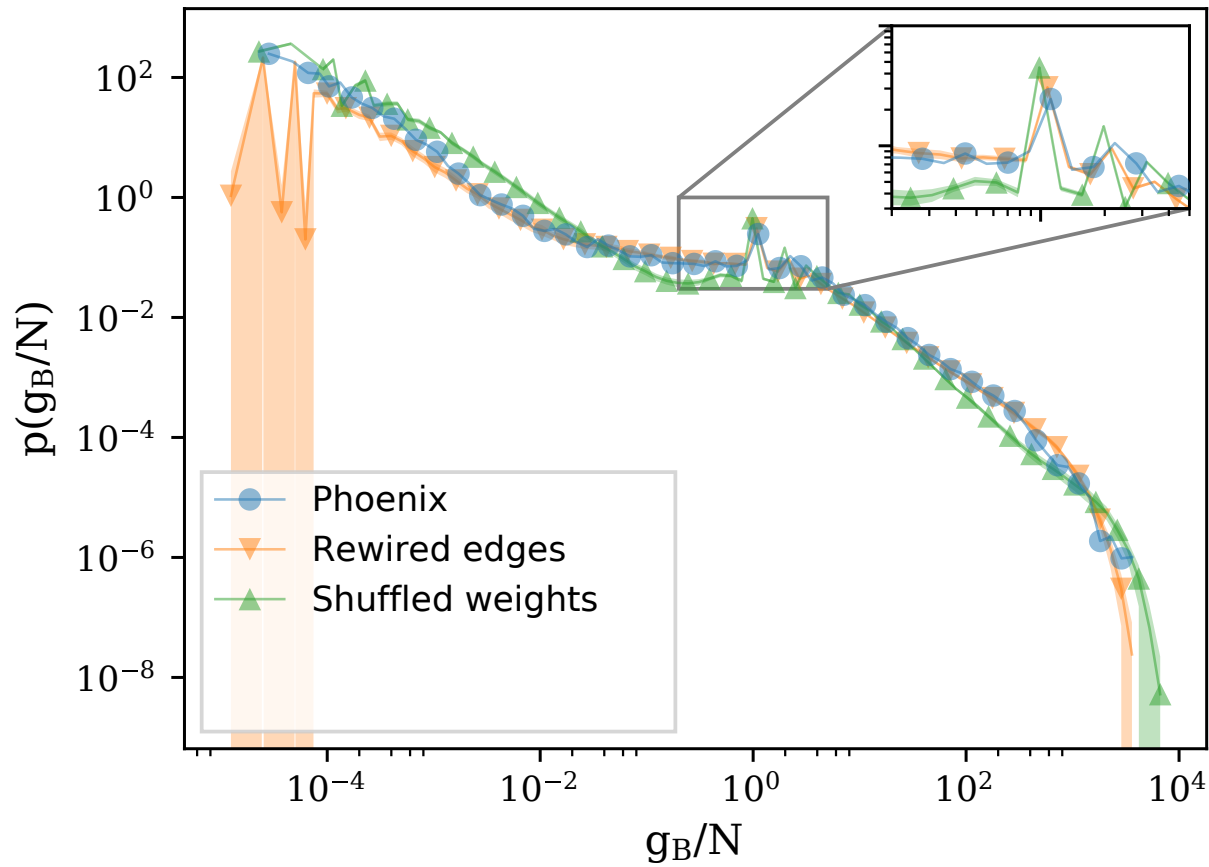




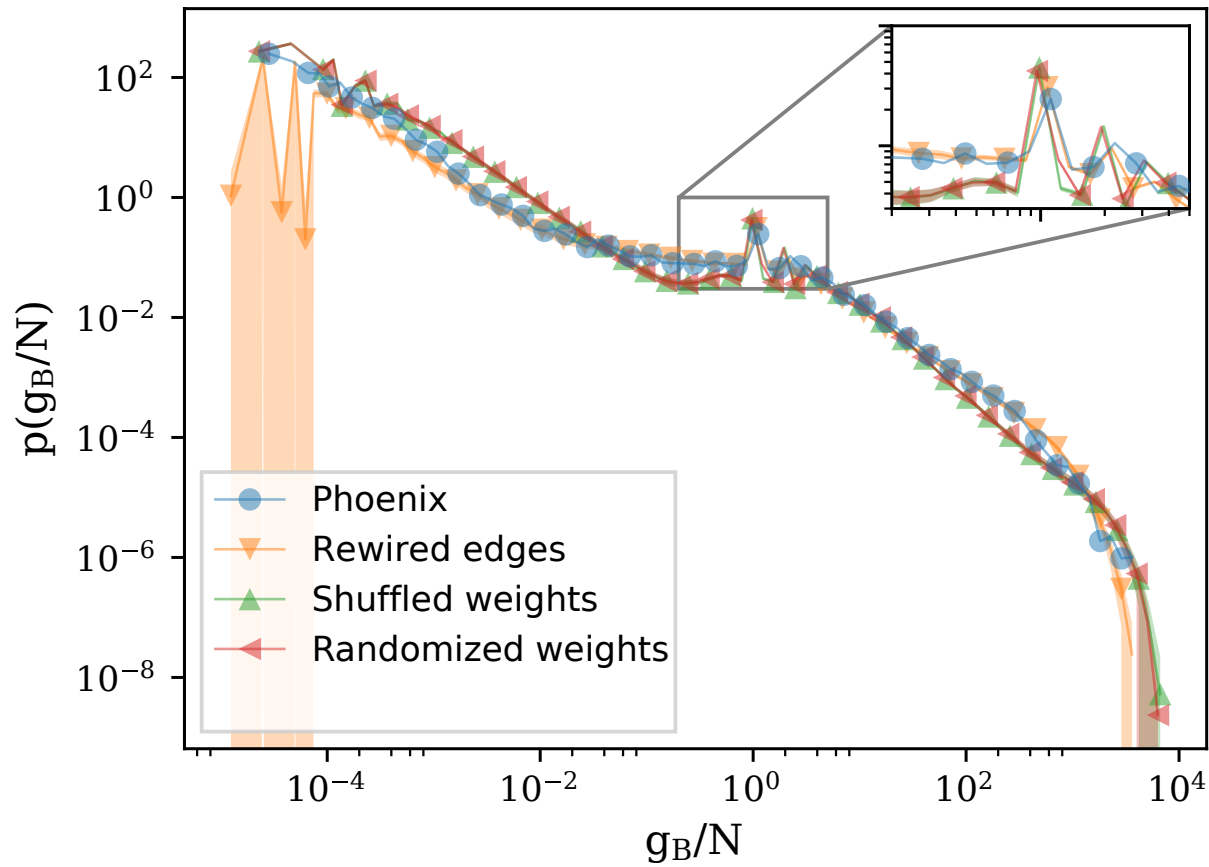
# Random planar graphs



# Random planar graphs

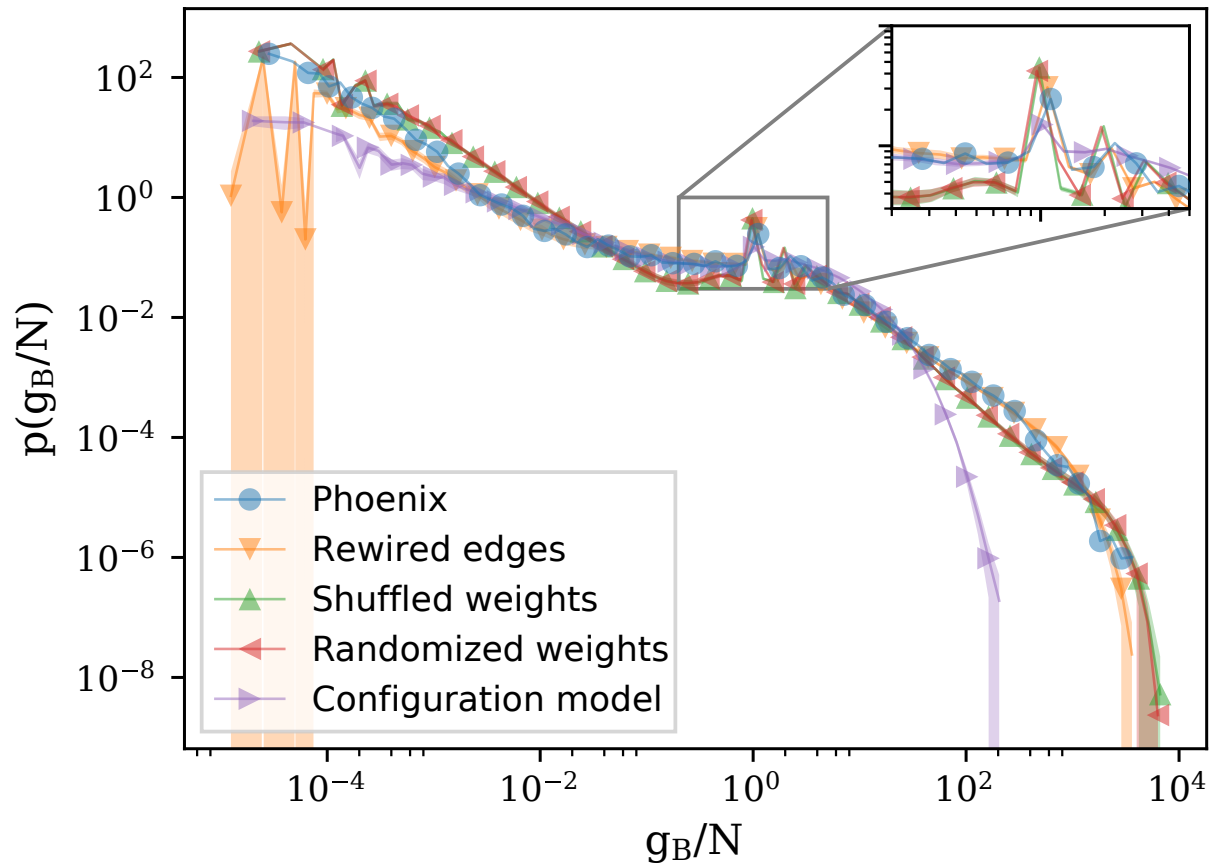


# Random planar graphs





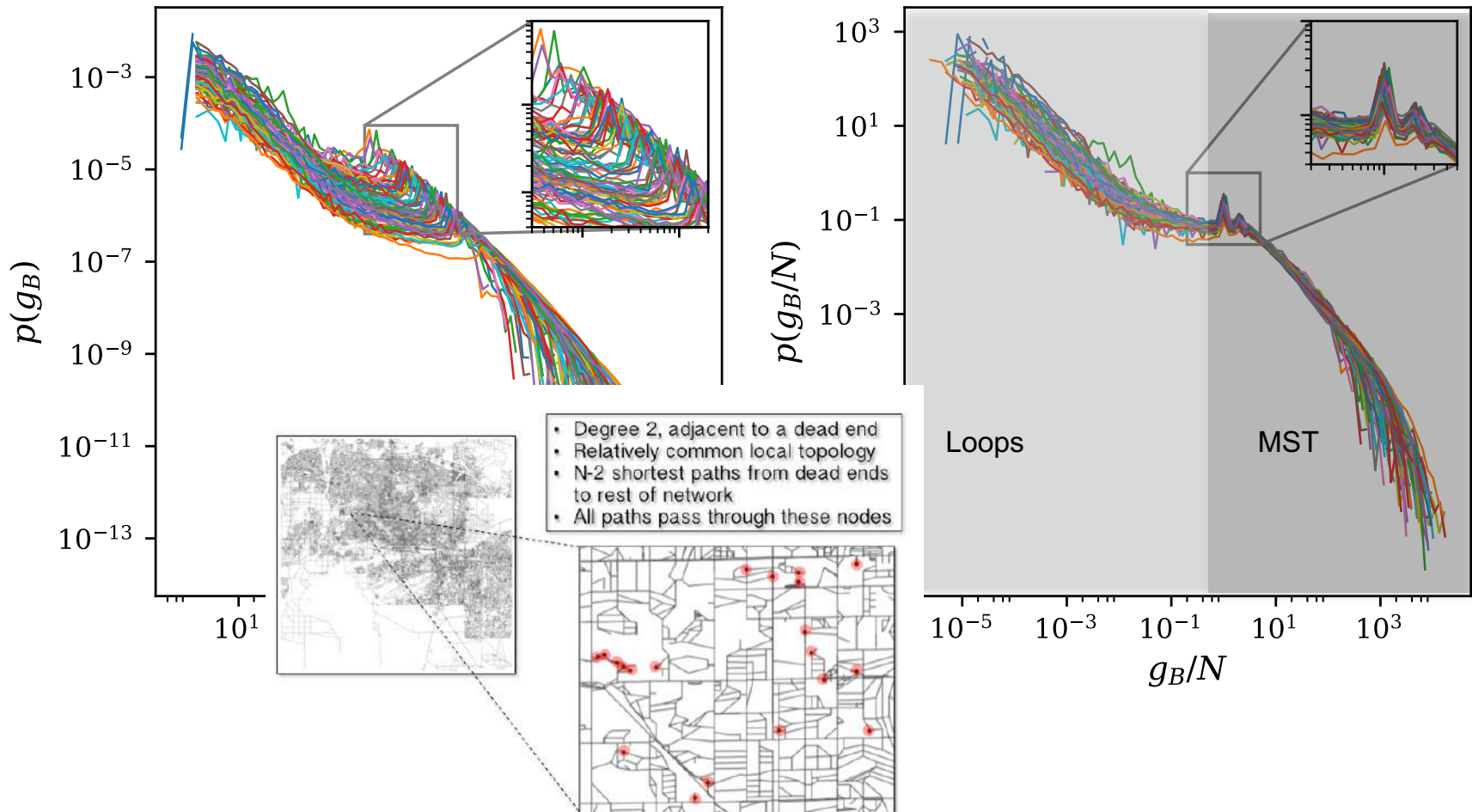
# Random planar graphs



$P(g_B)$  only specified by planarity, number of intersections  $N$  and roads  $e$ !

**Explanation for bimodality  
and Size-scaling?**

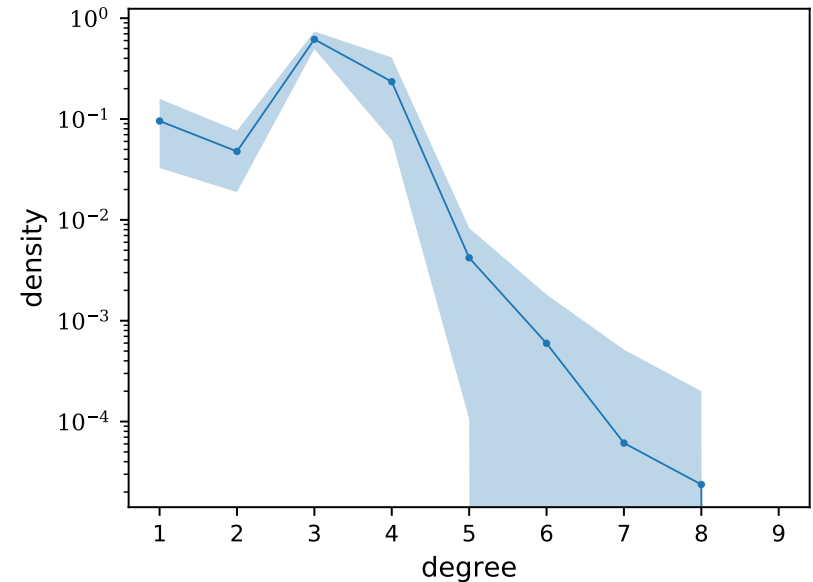
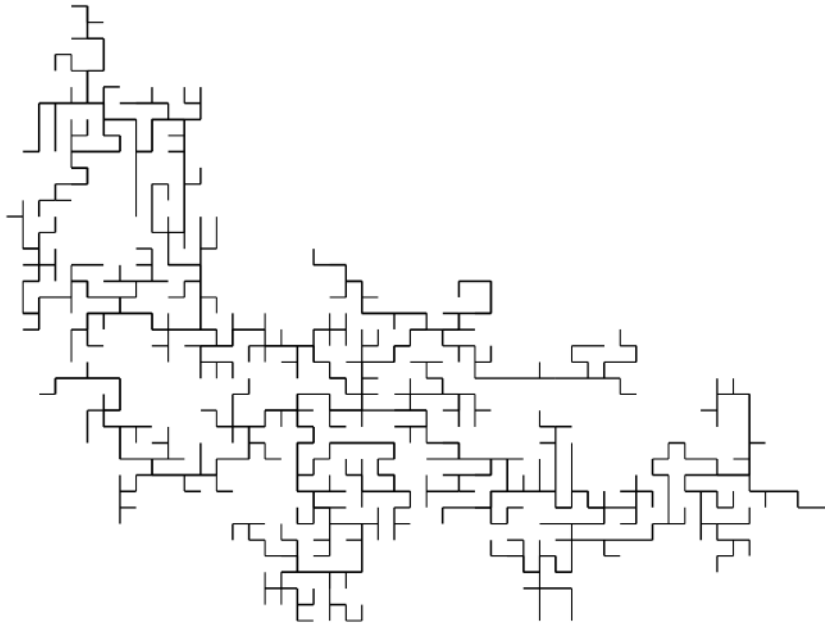
# Betweenness (1000 sq. miles)





# Quasi-analytical description

K-ary Tree (Cayley) approximation



Nodes  $N$

Vertex  $v$

Branching-ratio  $k$

Depth  $l$

Leaf-level  $L$

$$g_b(v|k, l) \sim O(Nk^{L-l})$$

$$P(g_b(v|k, l)) \sim \frac{k^l}{N}$$

$$P(g_b/N) \sim \left(\frac{g_b}{N}\right)^{-1-\frac{1}{L-l}}$$

**How does one explain the  
low betweenness regime?**

**Effect of loops**

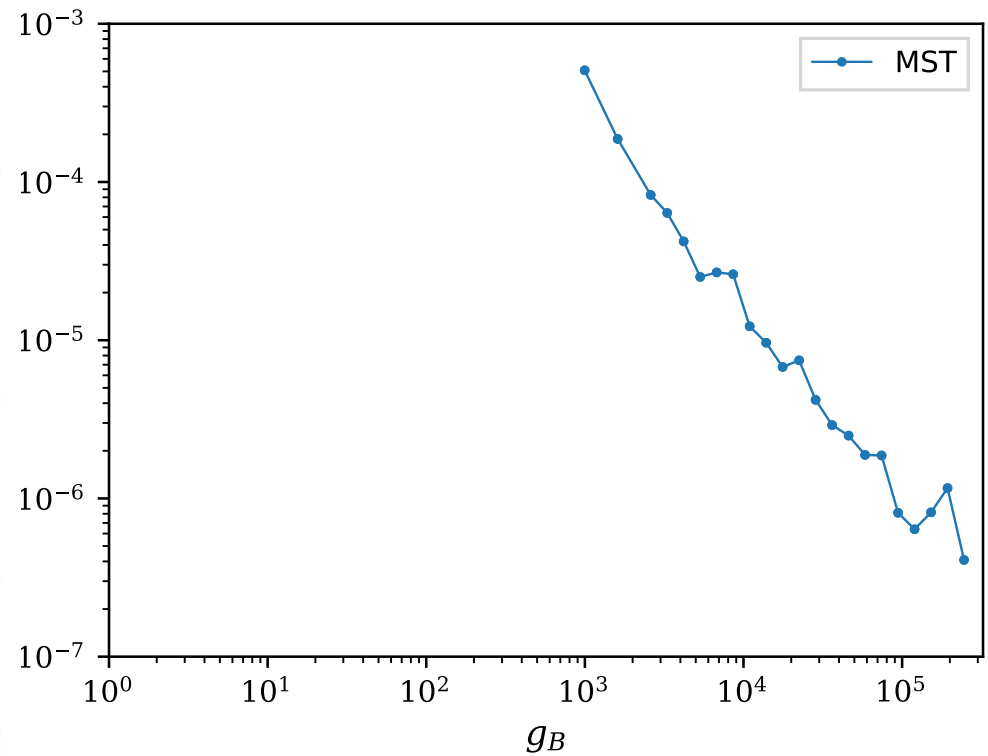
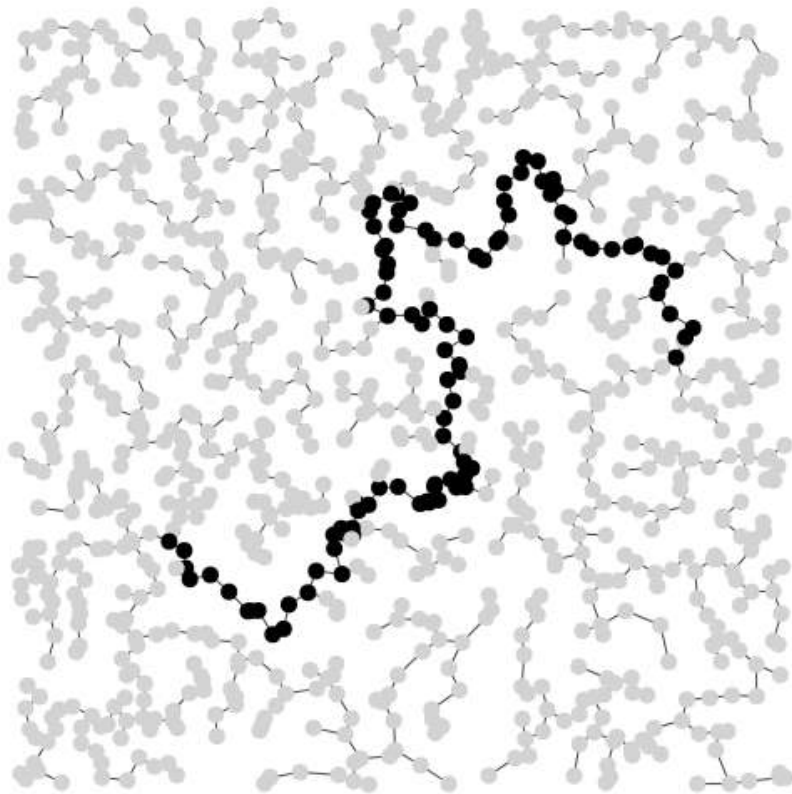
# Effect of Edge-Density

Euler's formula

$$N - e + f = 2$$

Edge Density

$$d_e = \frac{e}{e_{DT}}$$





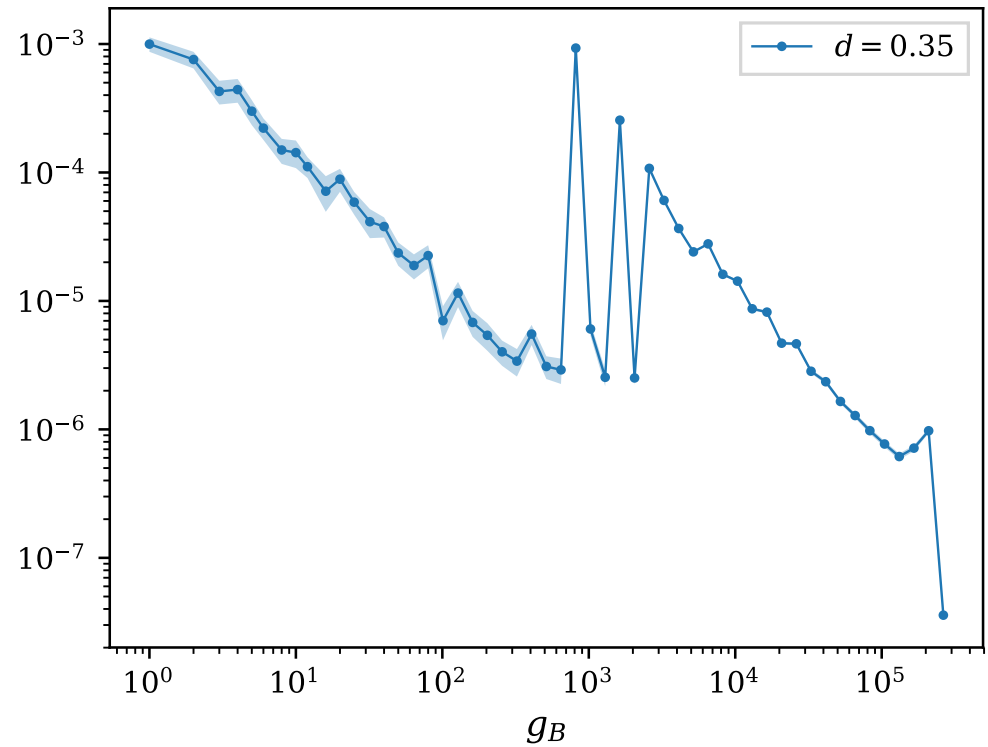
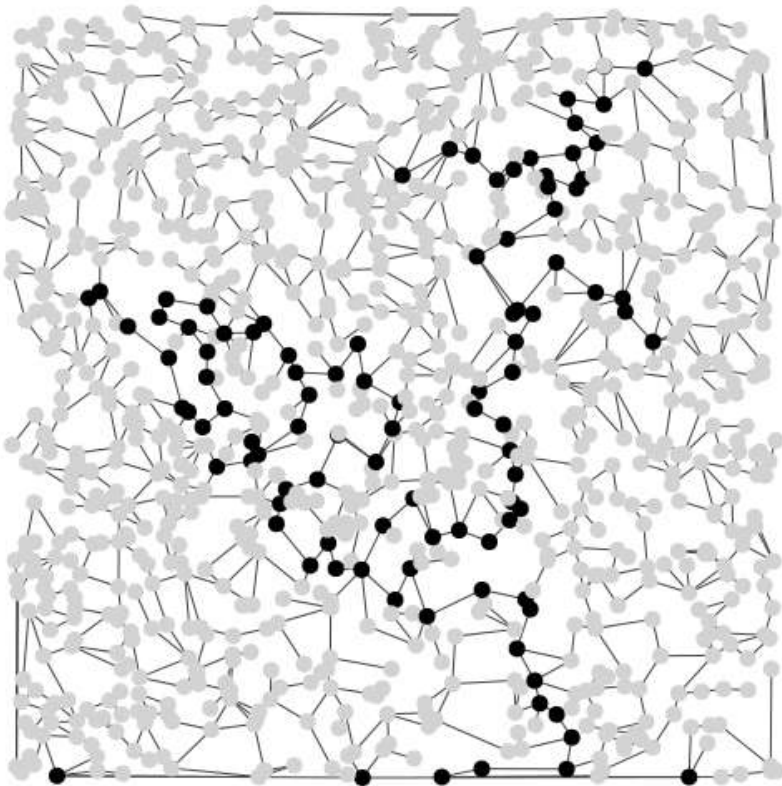
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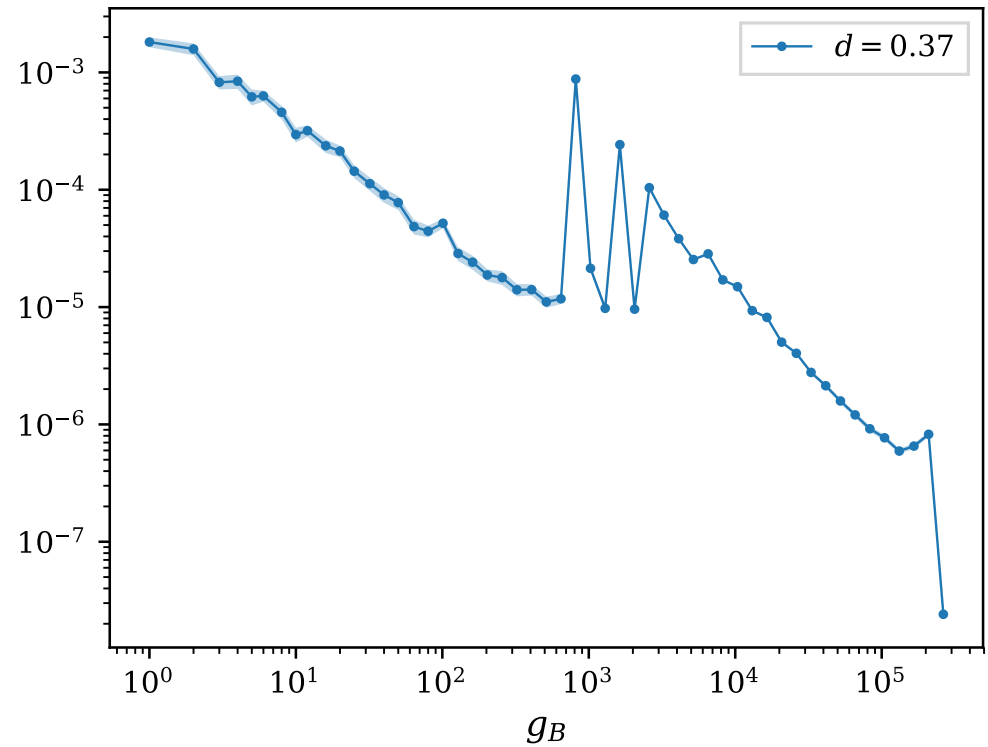
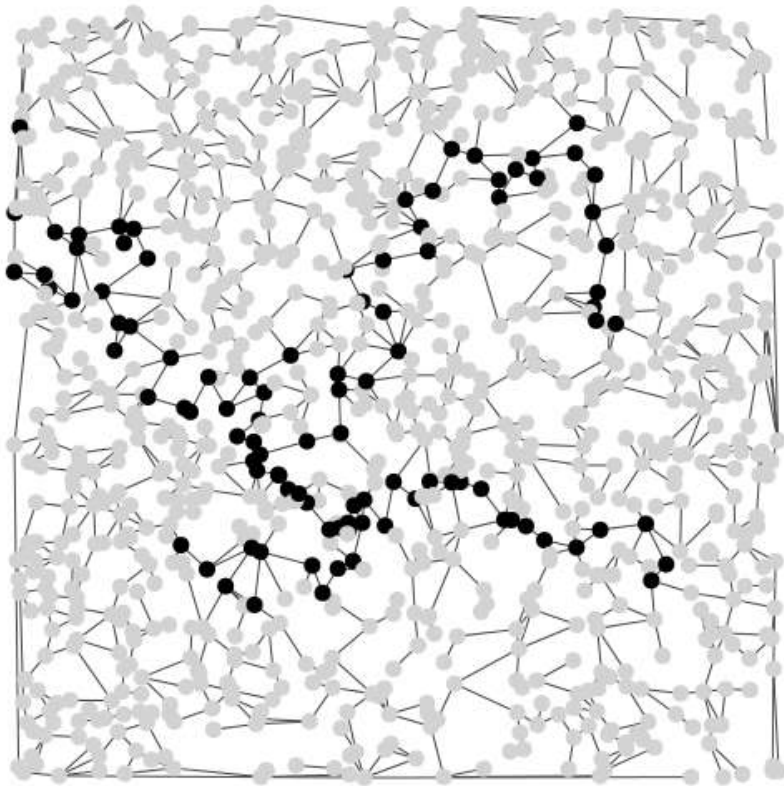
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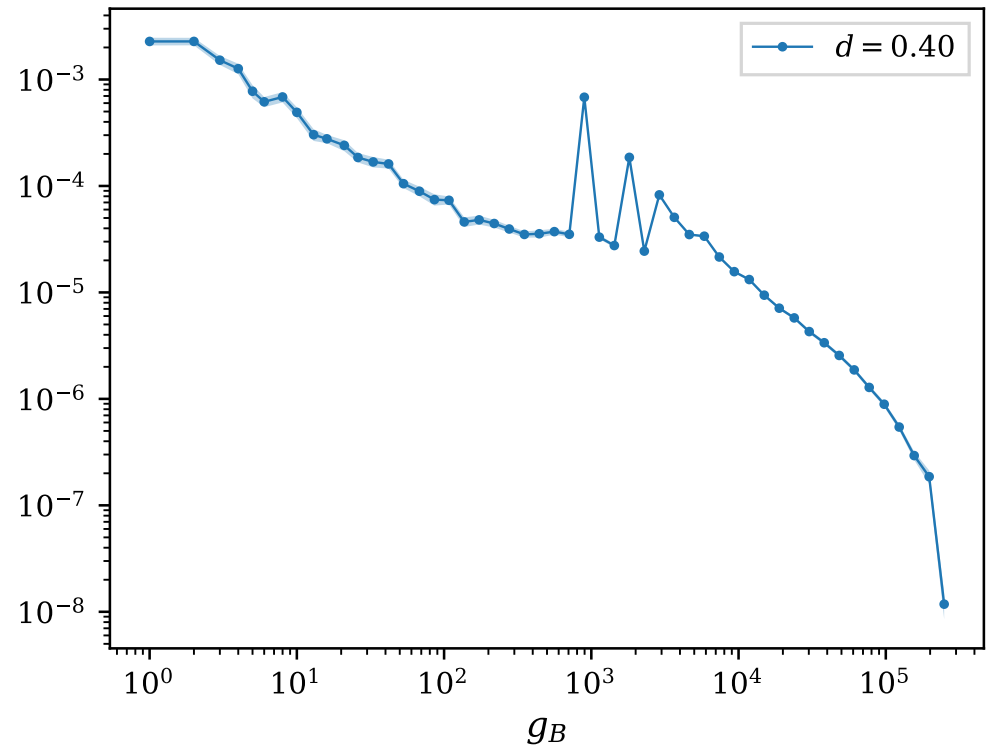
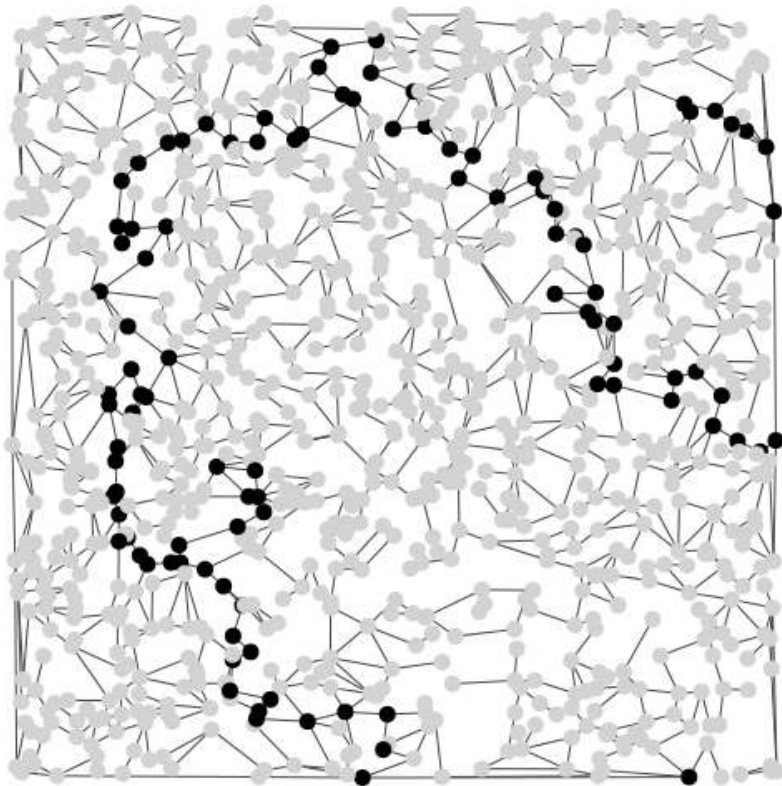
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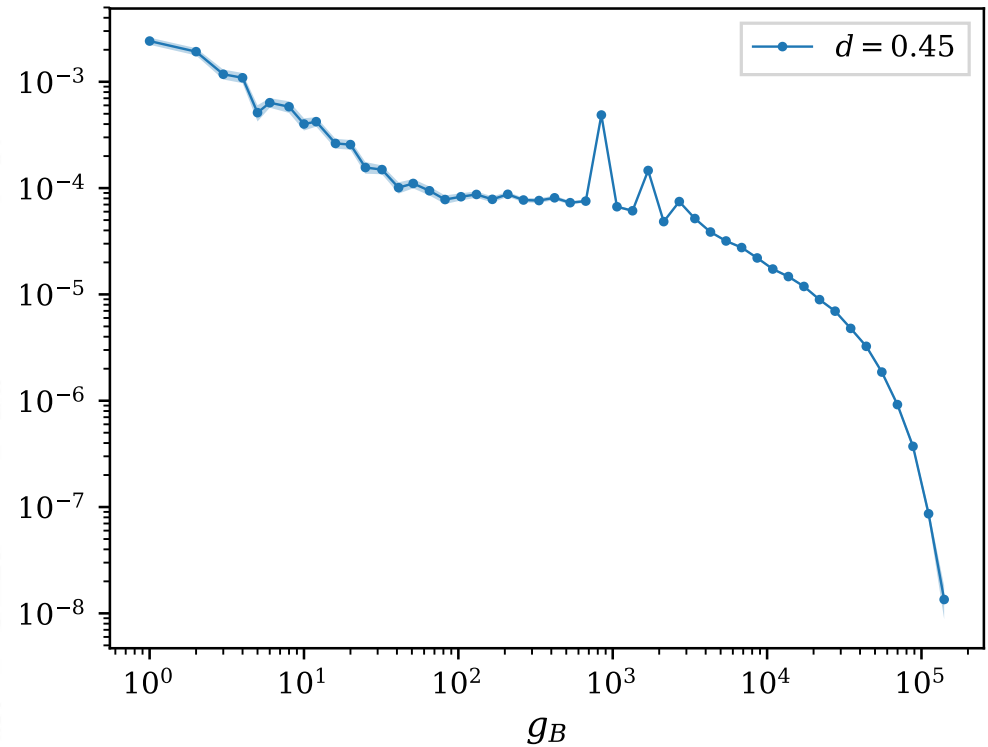
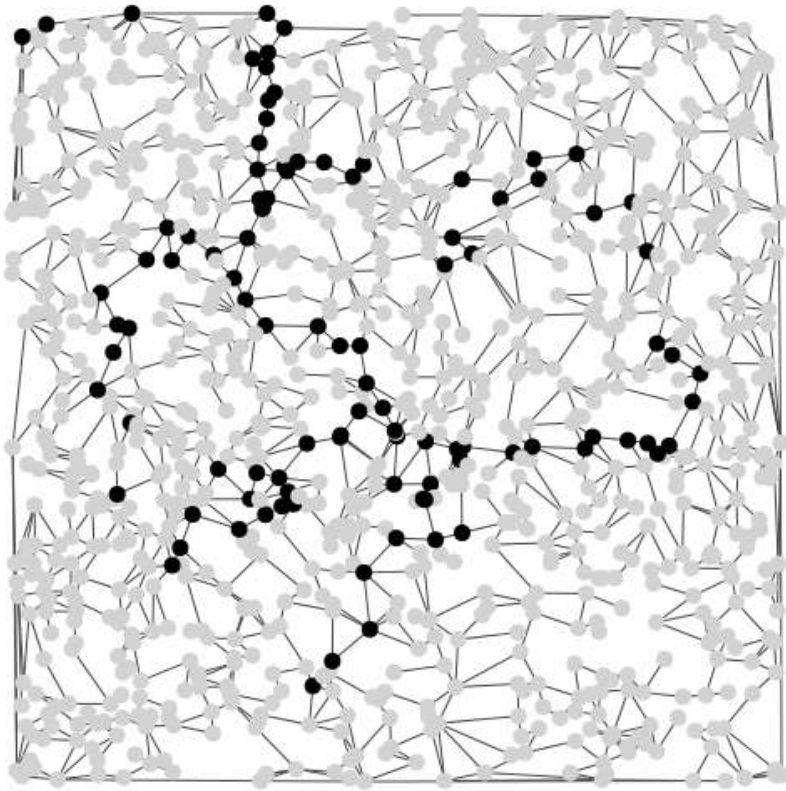
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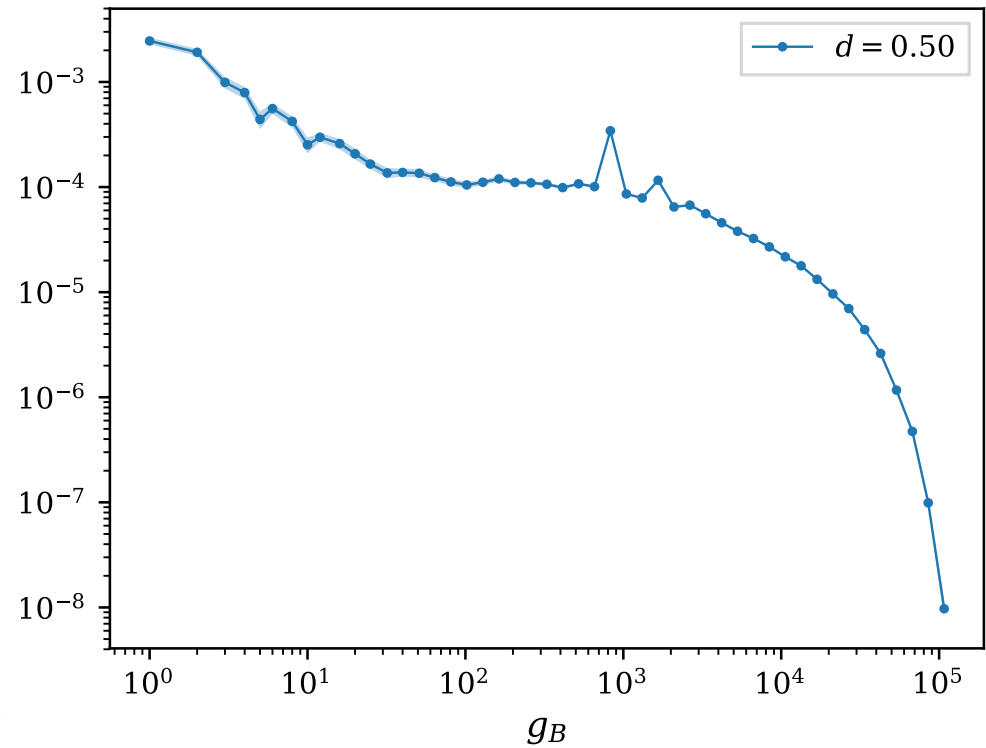
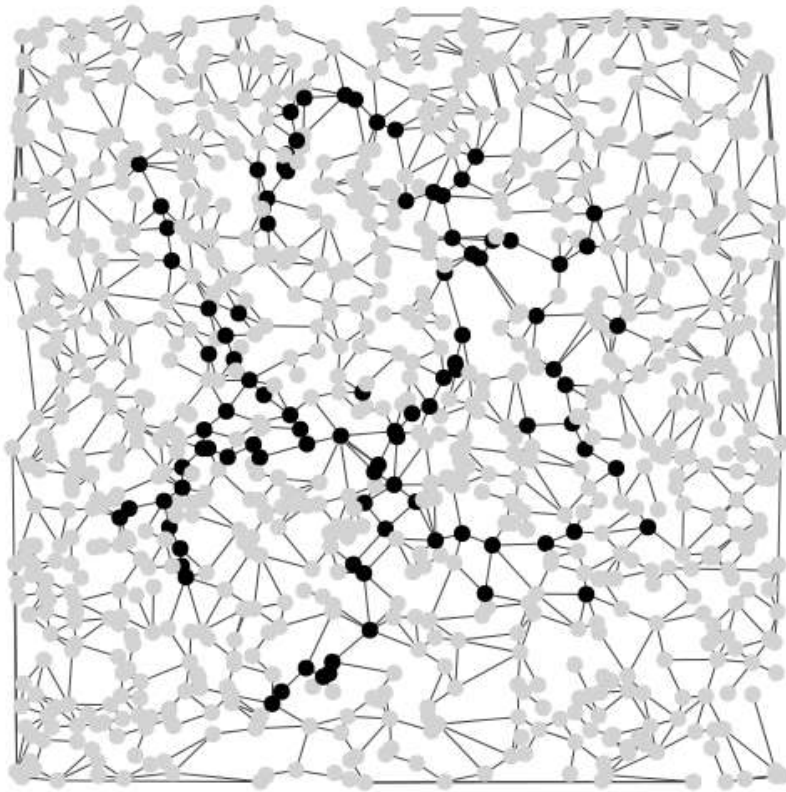
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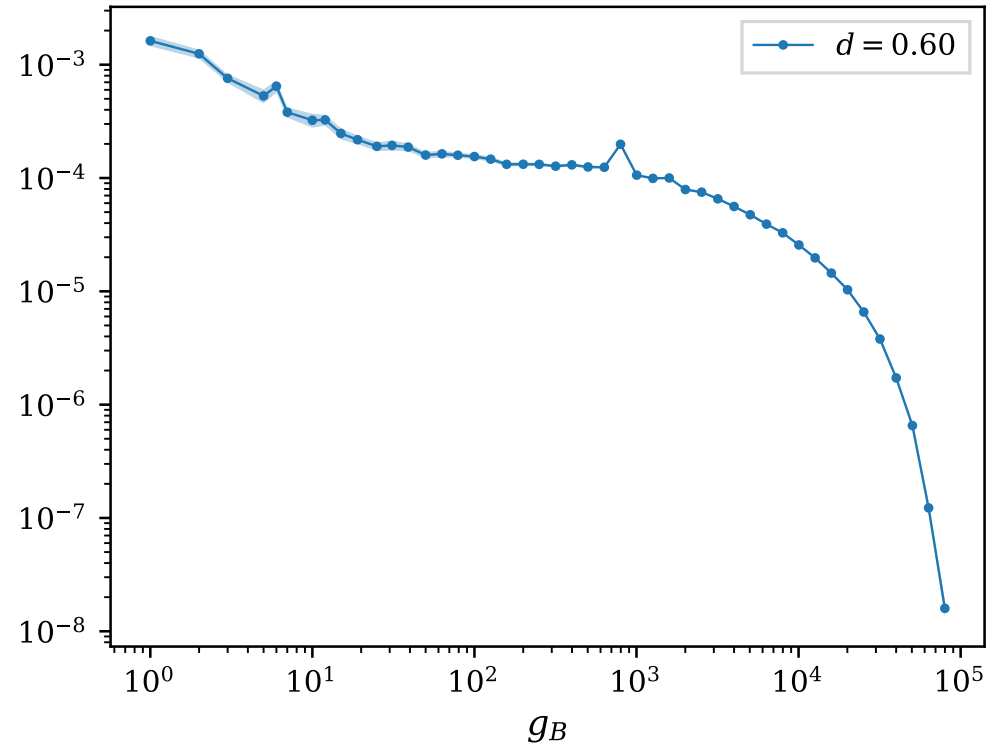
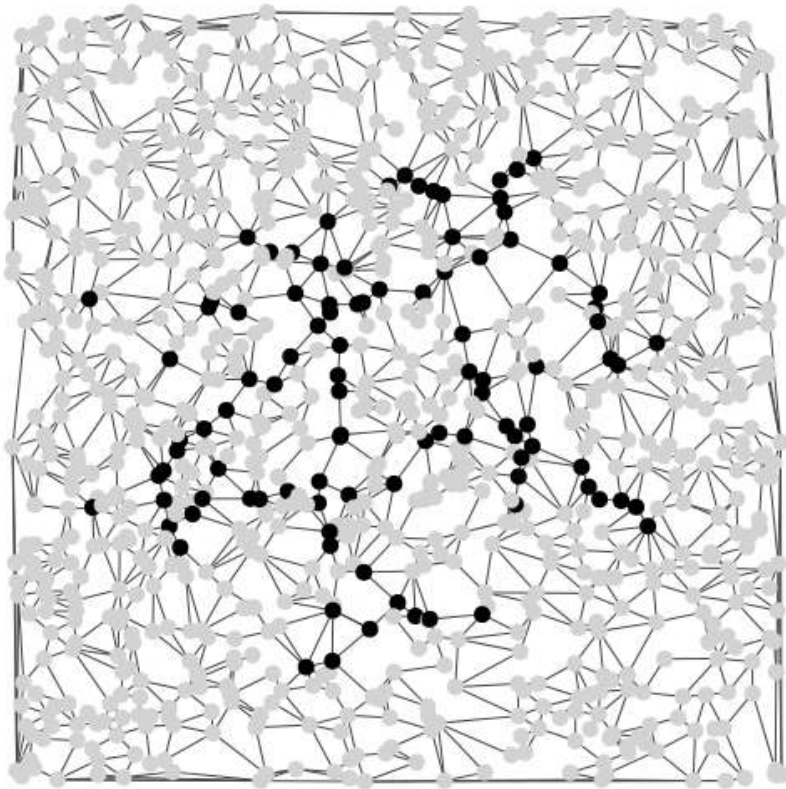
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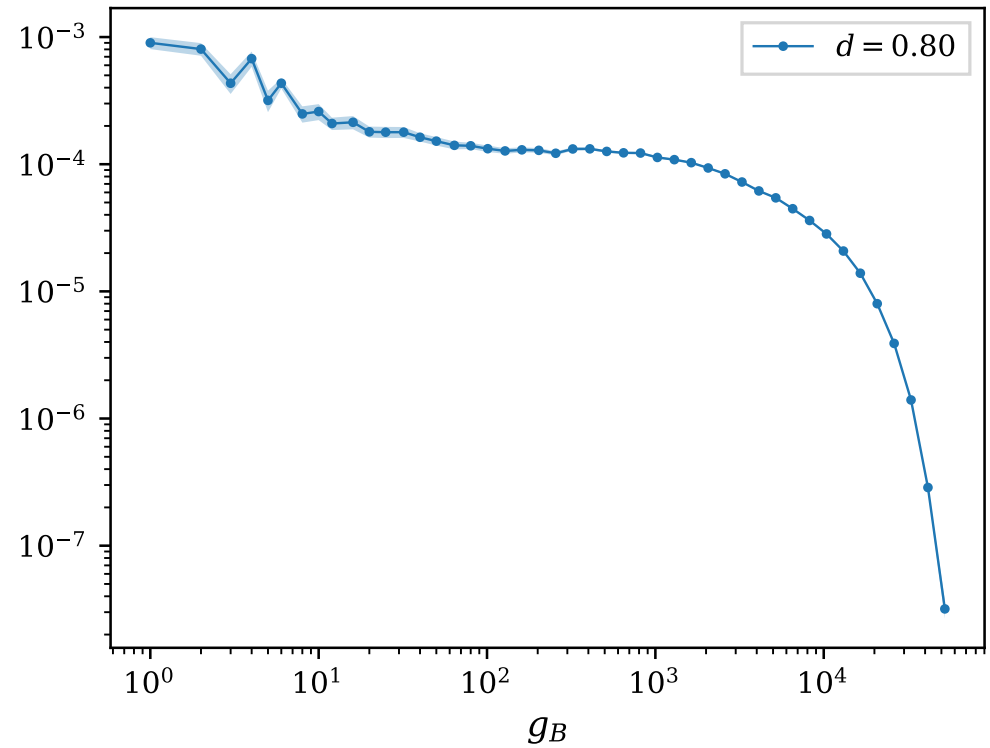
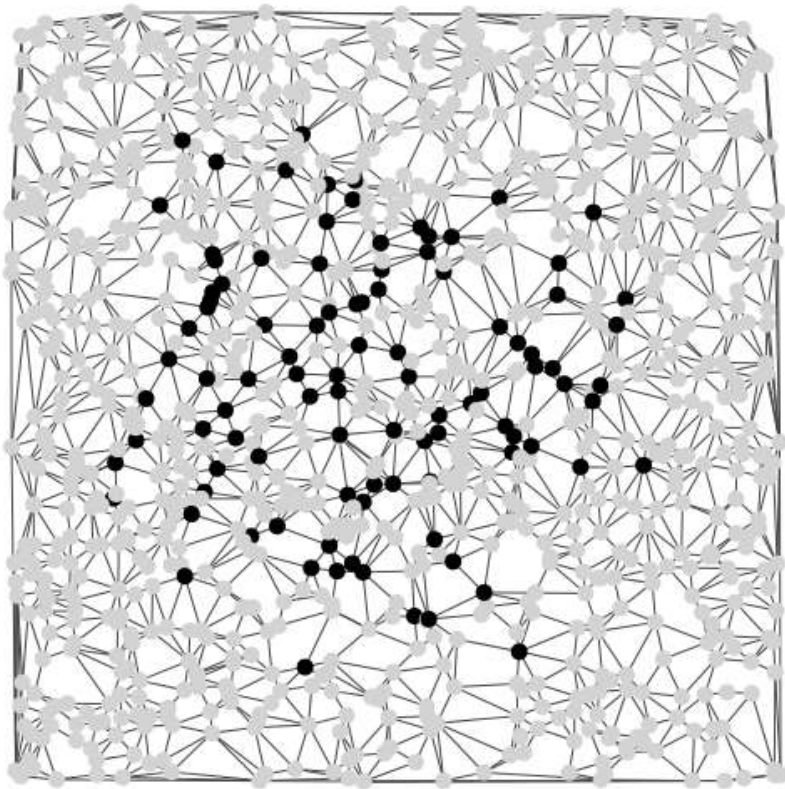
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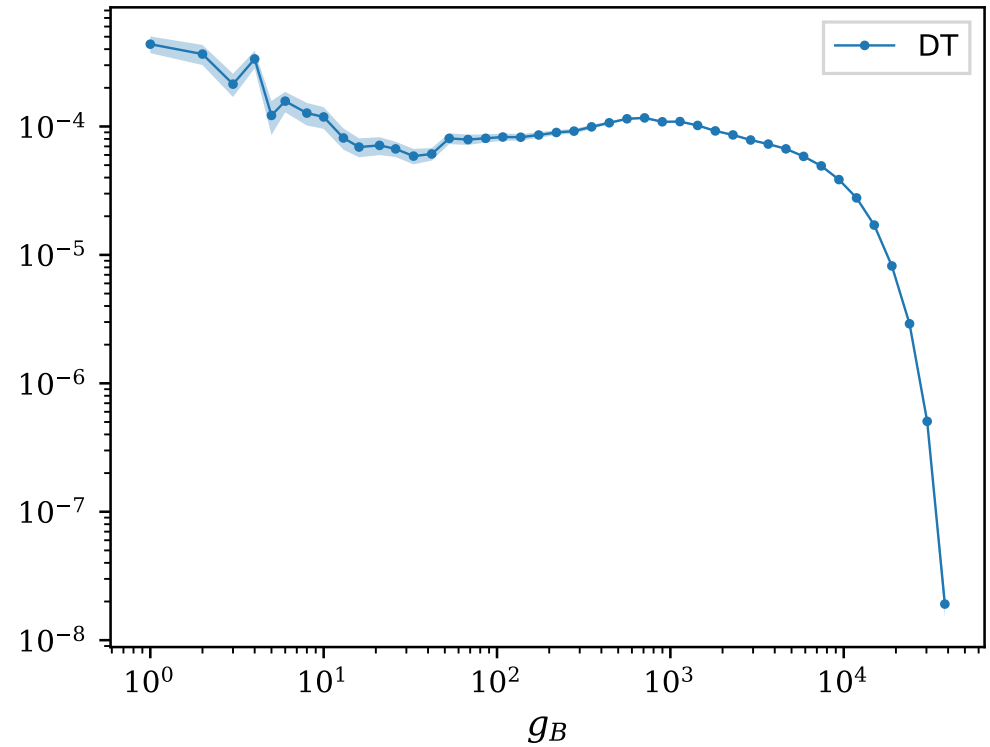
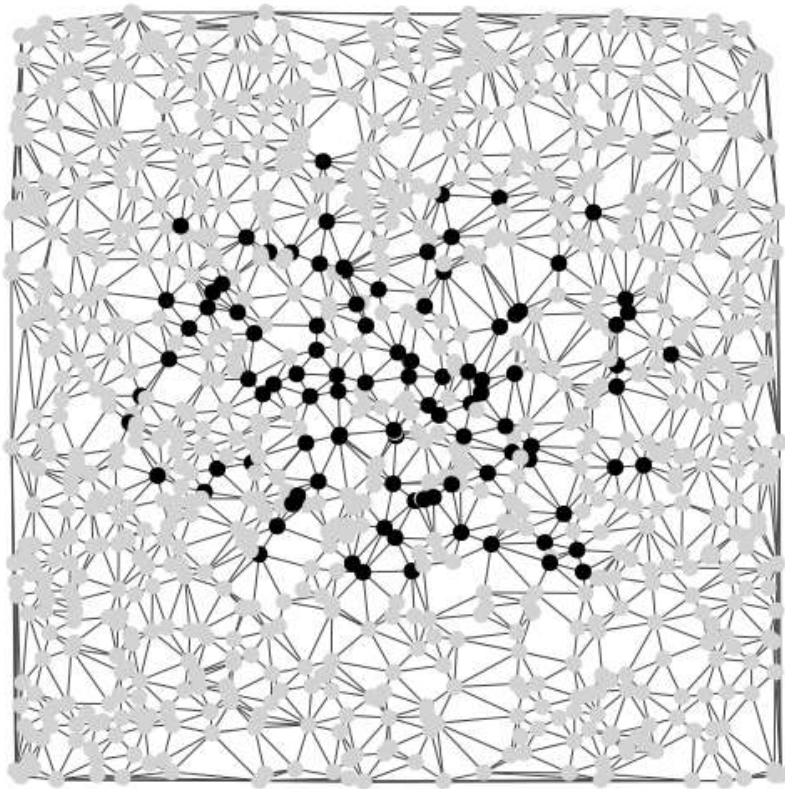
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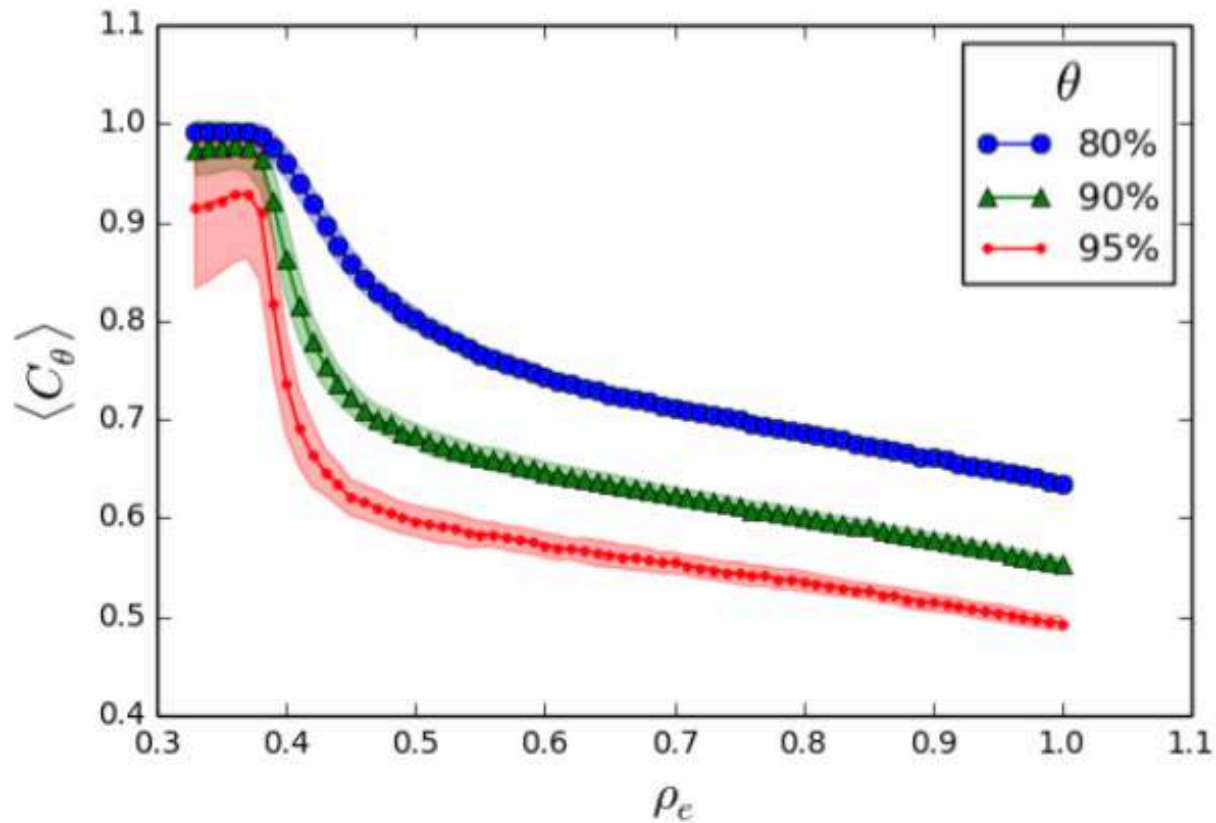
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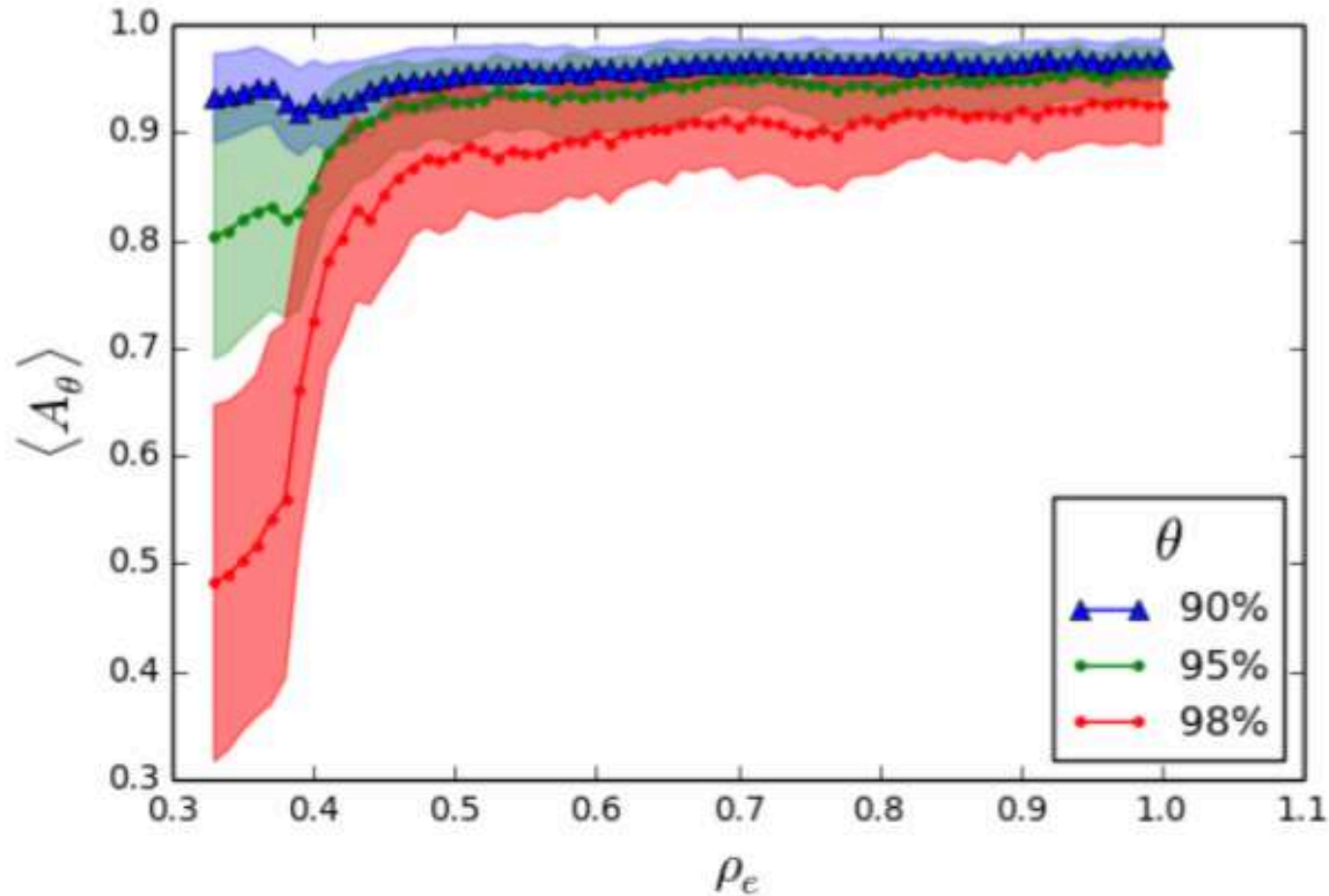


# Spatial Clustering



$$C_\theta = \frac{1}{\langle w \rangle N_\theta} \sum_{i=1}^{N_\theta} \| \mathbf{x}_i - \mathbf{x}_{\text{cm}} \|^2; \quad \mathbf{x}_{\text{cm}} = N_\theta^{-1} \sum_{i=1}^{N_\theta} \mathbf{x}_i$$

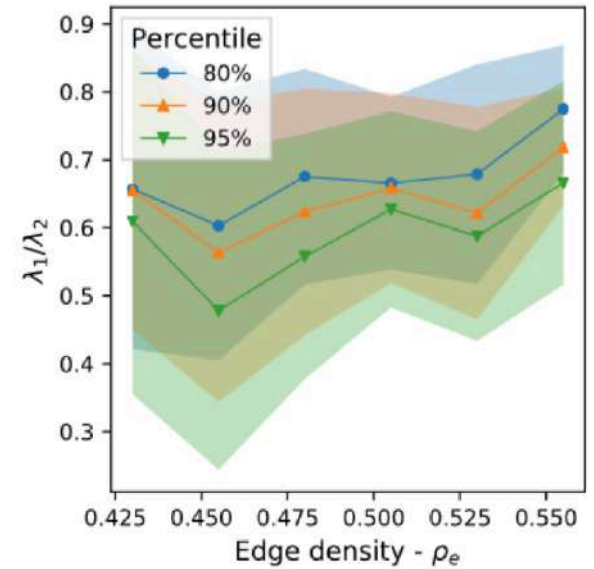
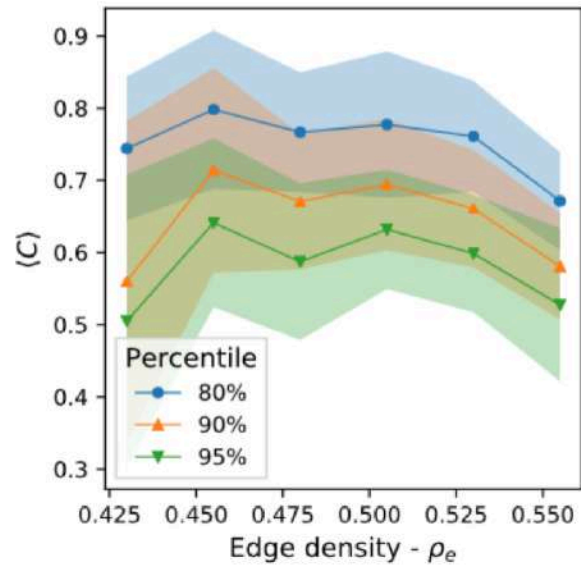
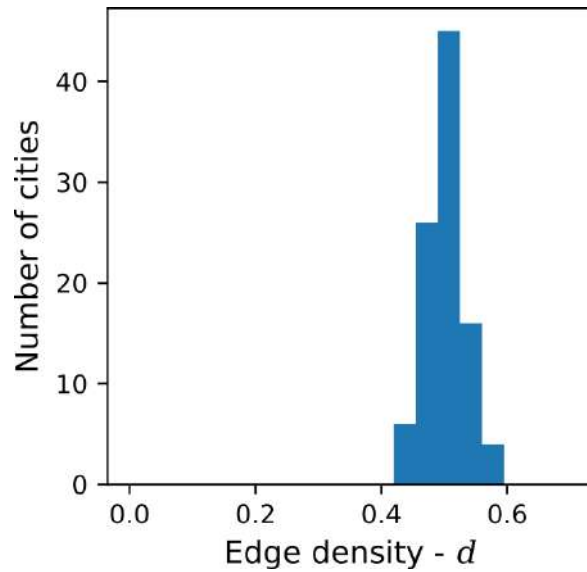
# Measure of anisotropy



Eigenvalue ratio of  
covariance matrix:

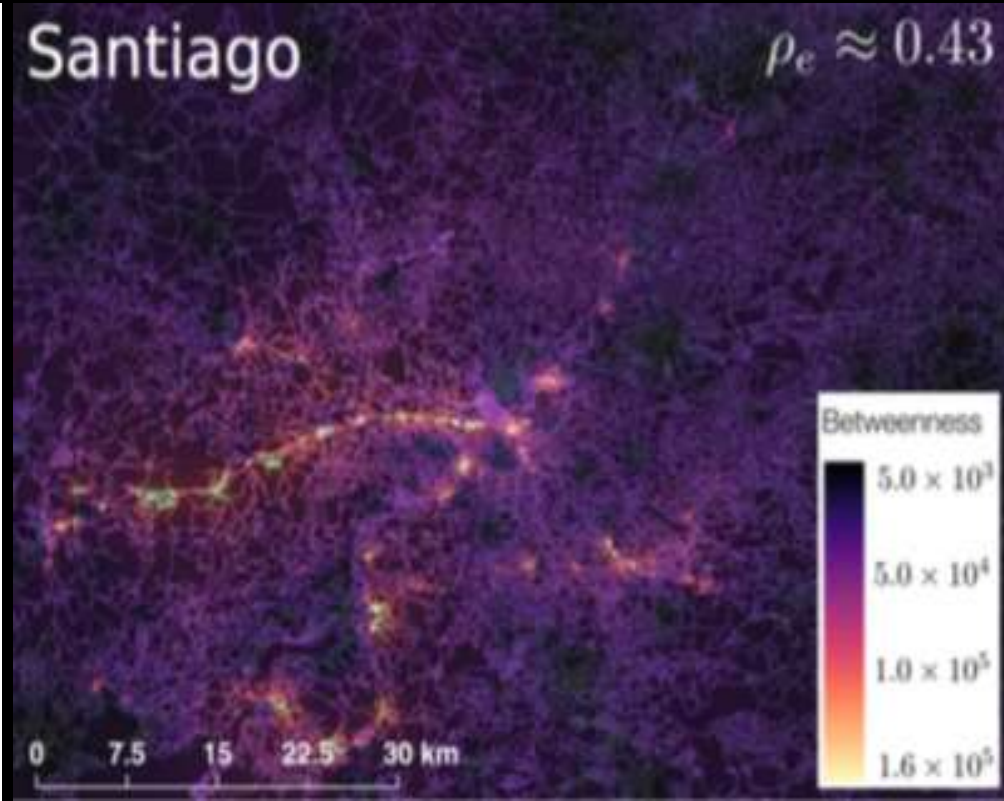
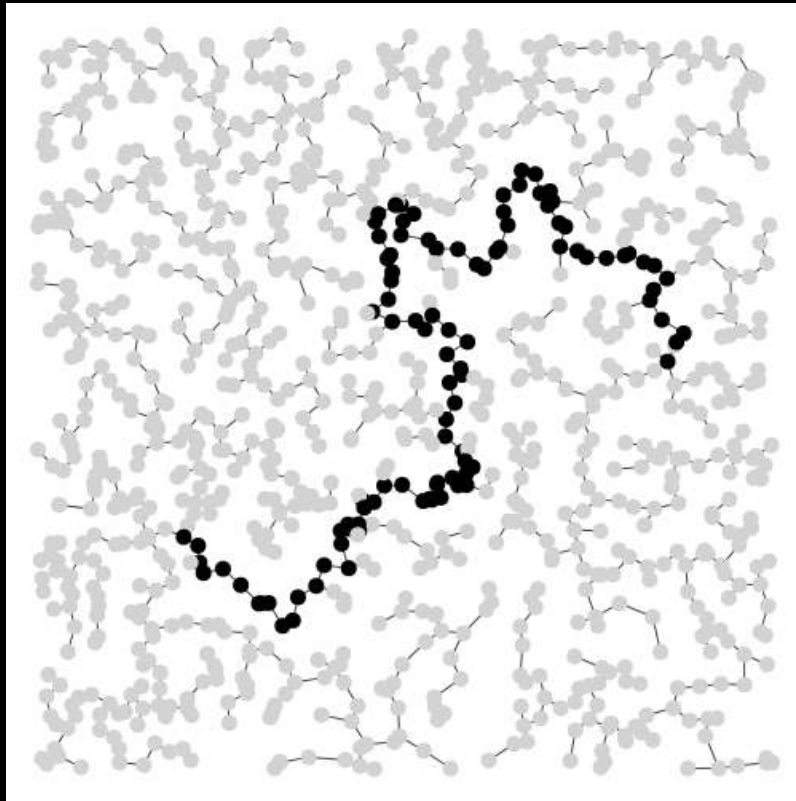
$$A_\theta = \frac{\lambda_1}{\lambda_2}; \quad \lambda_1 > \lambda_2$$

# Spatial Clustering and anisotropy in real cities

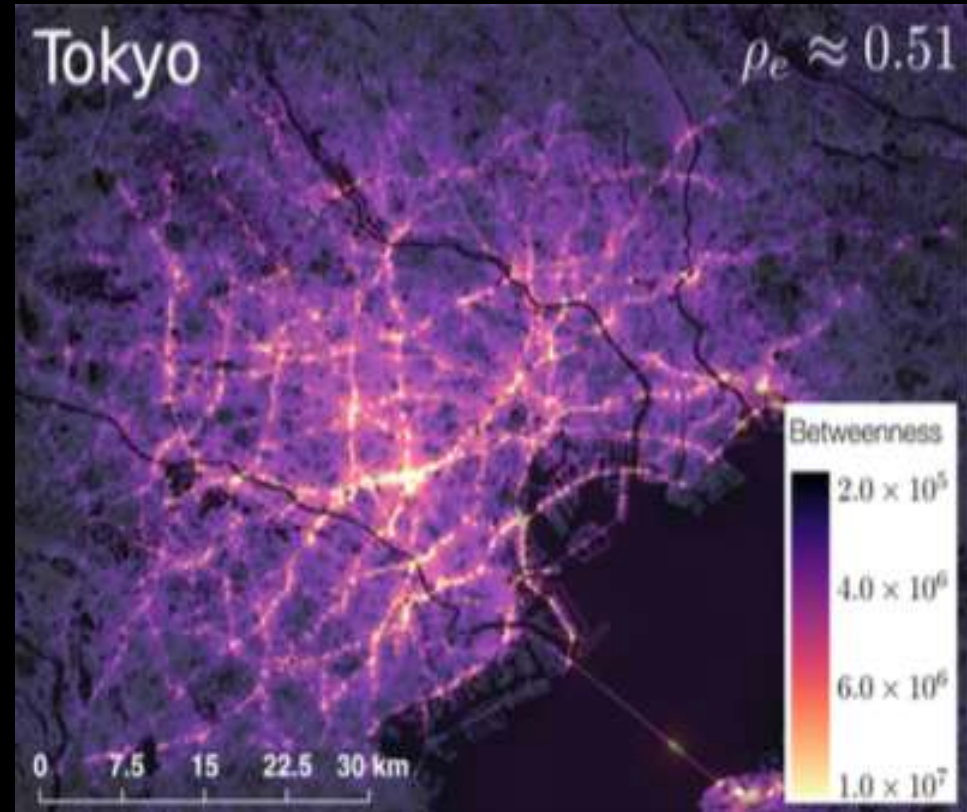
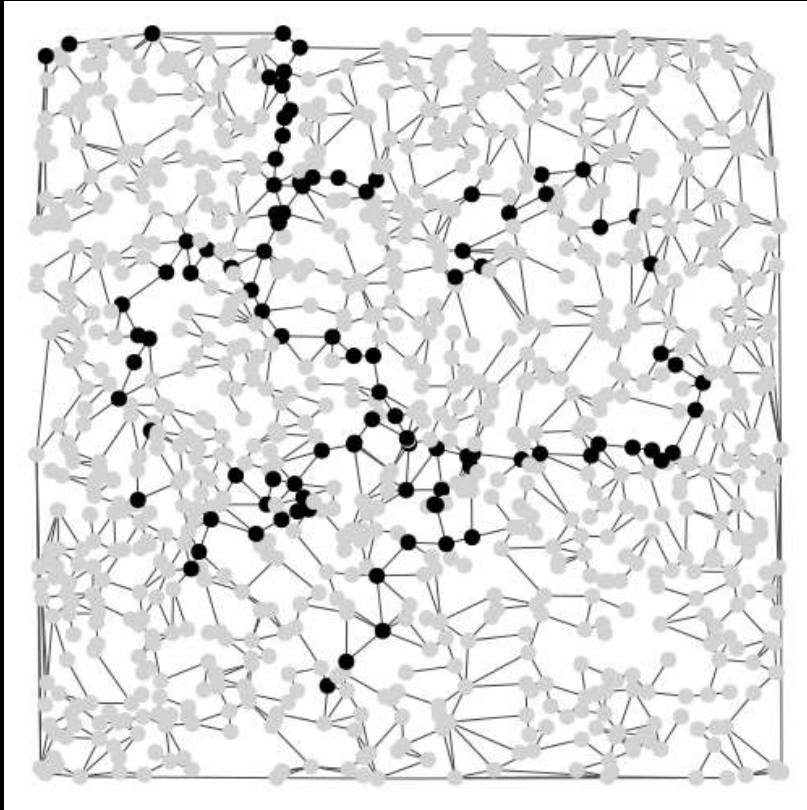




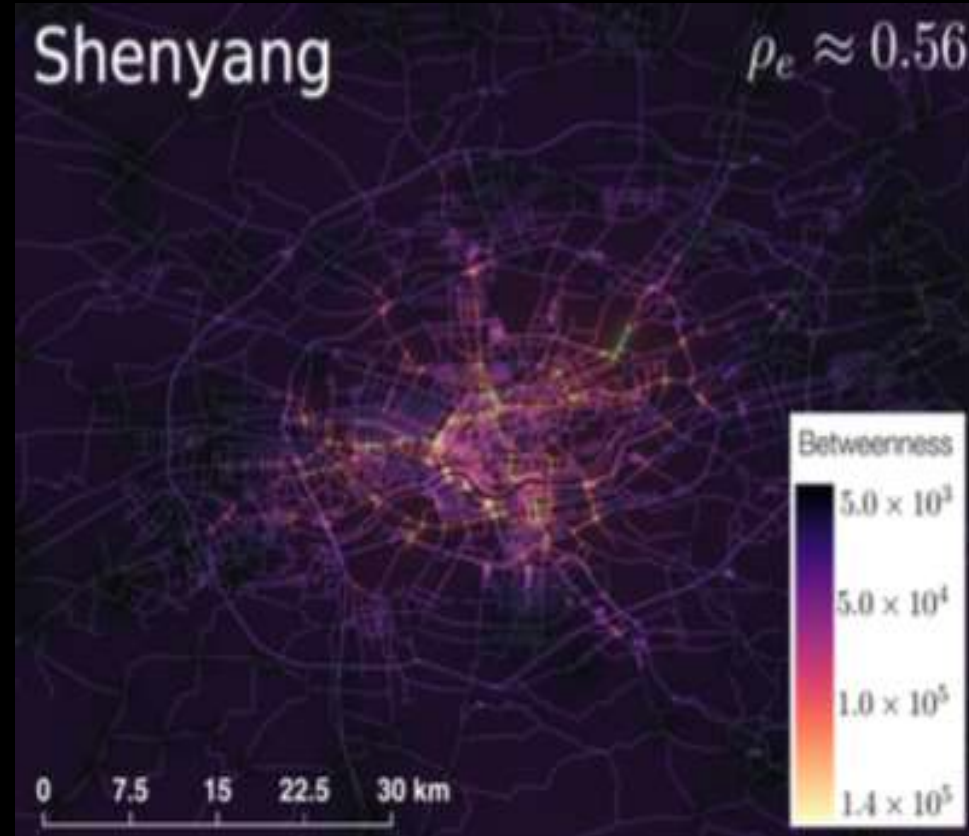
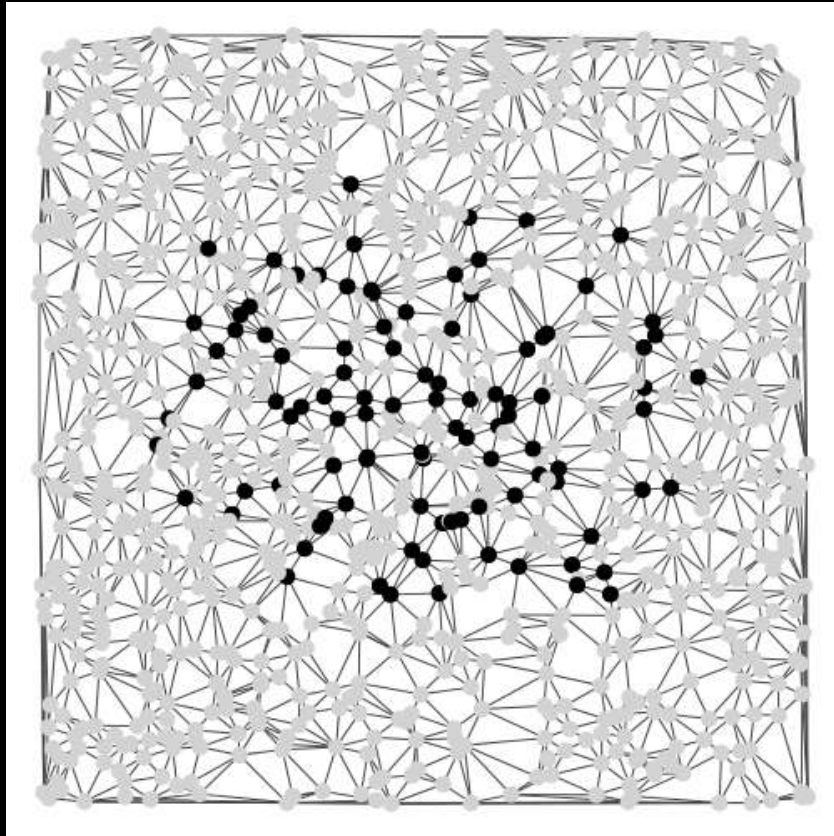
# Archetypes in real cities



# Archetypes in real cities



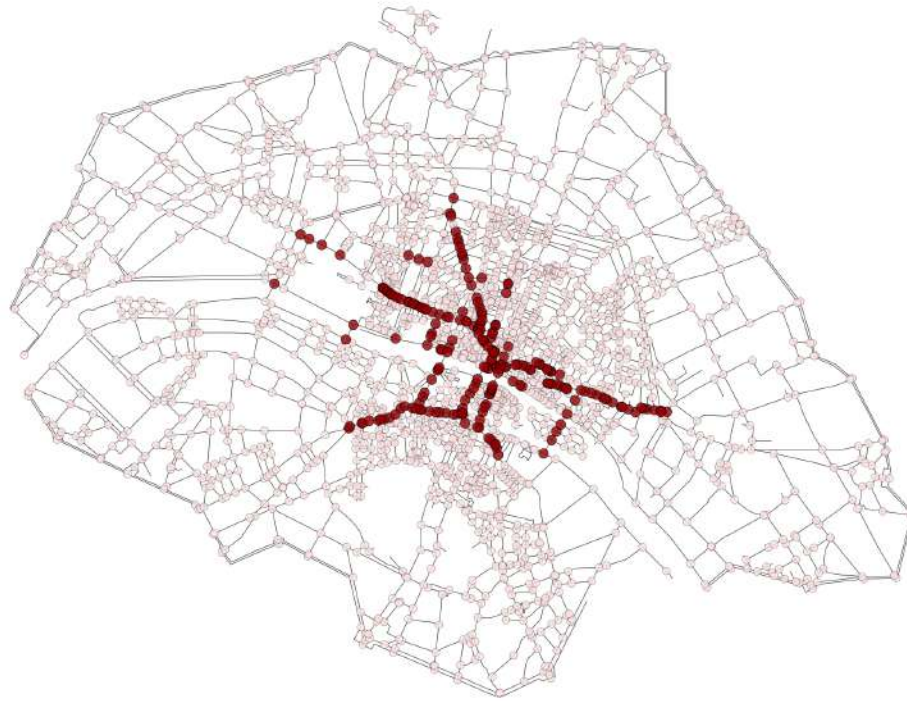
# Archetypes in real cities





# Evolution of Paris

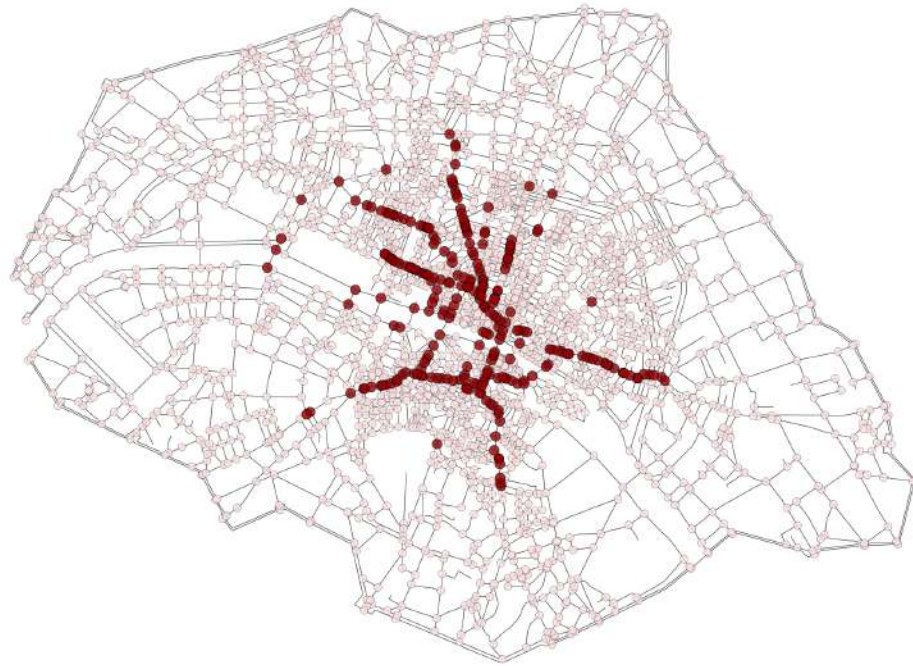
1790





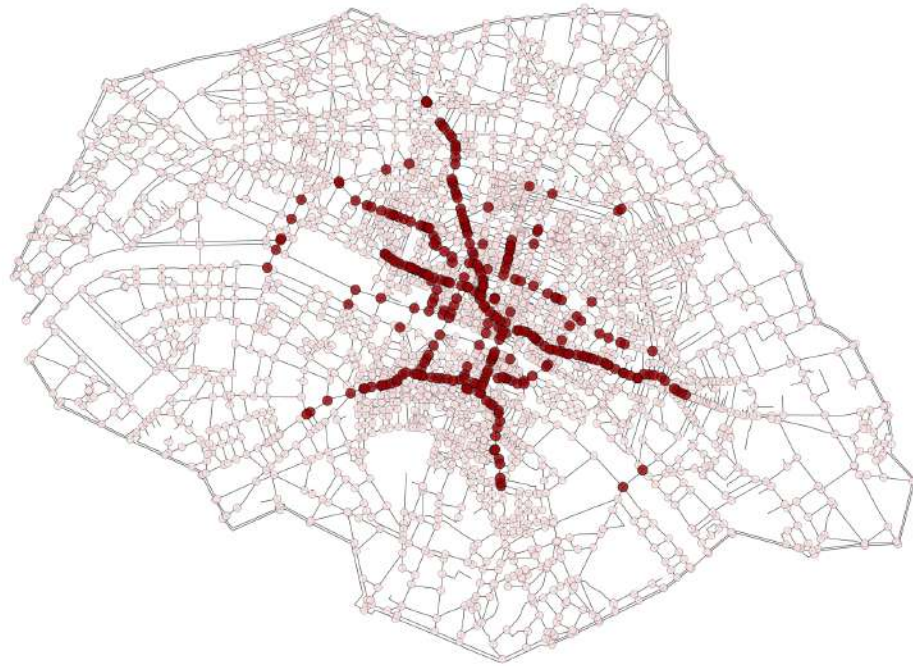
# Evolution of Paris

1836



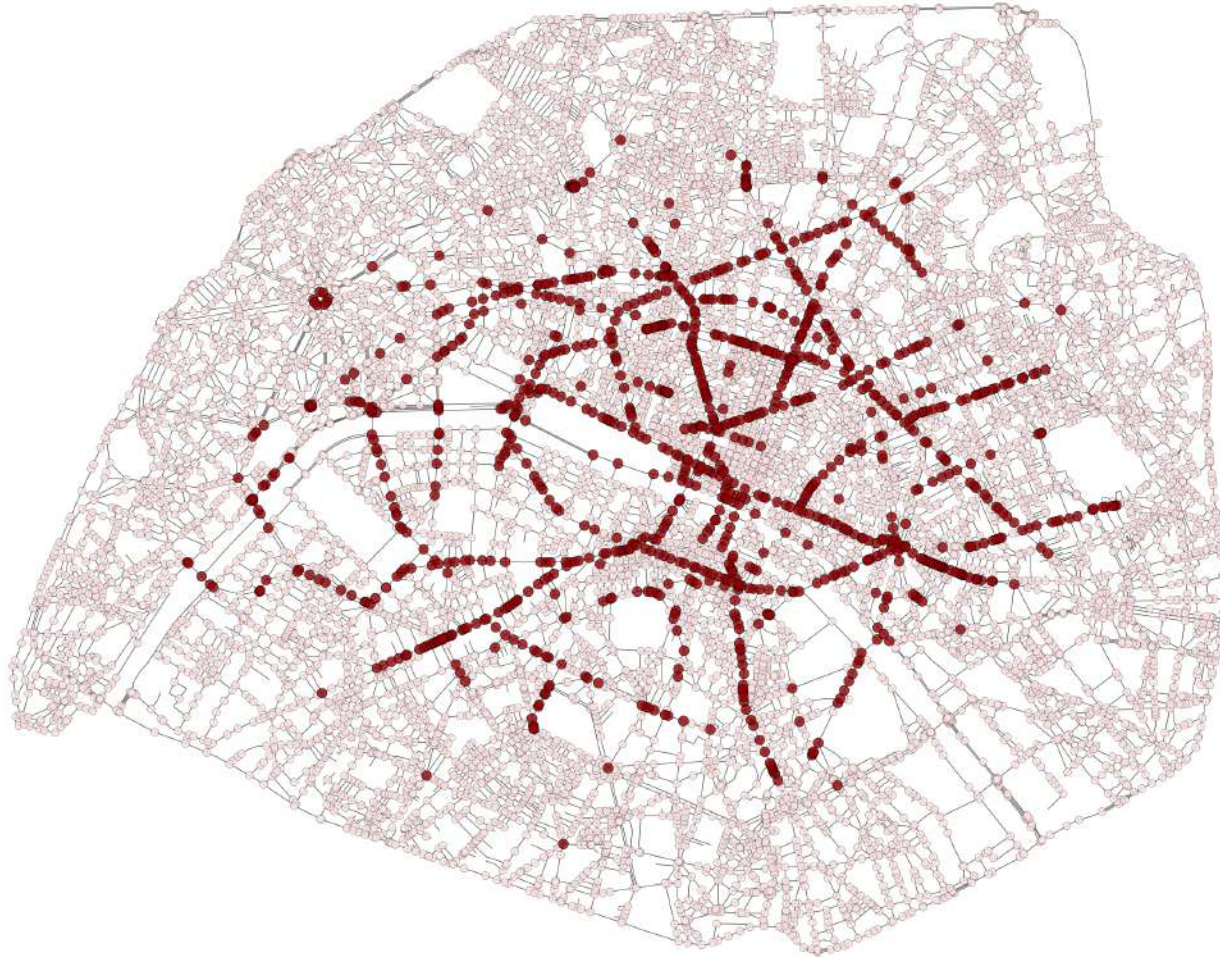
# Evolution of Paris

1849



# Evolution of Paris

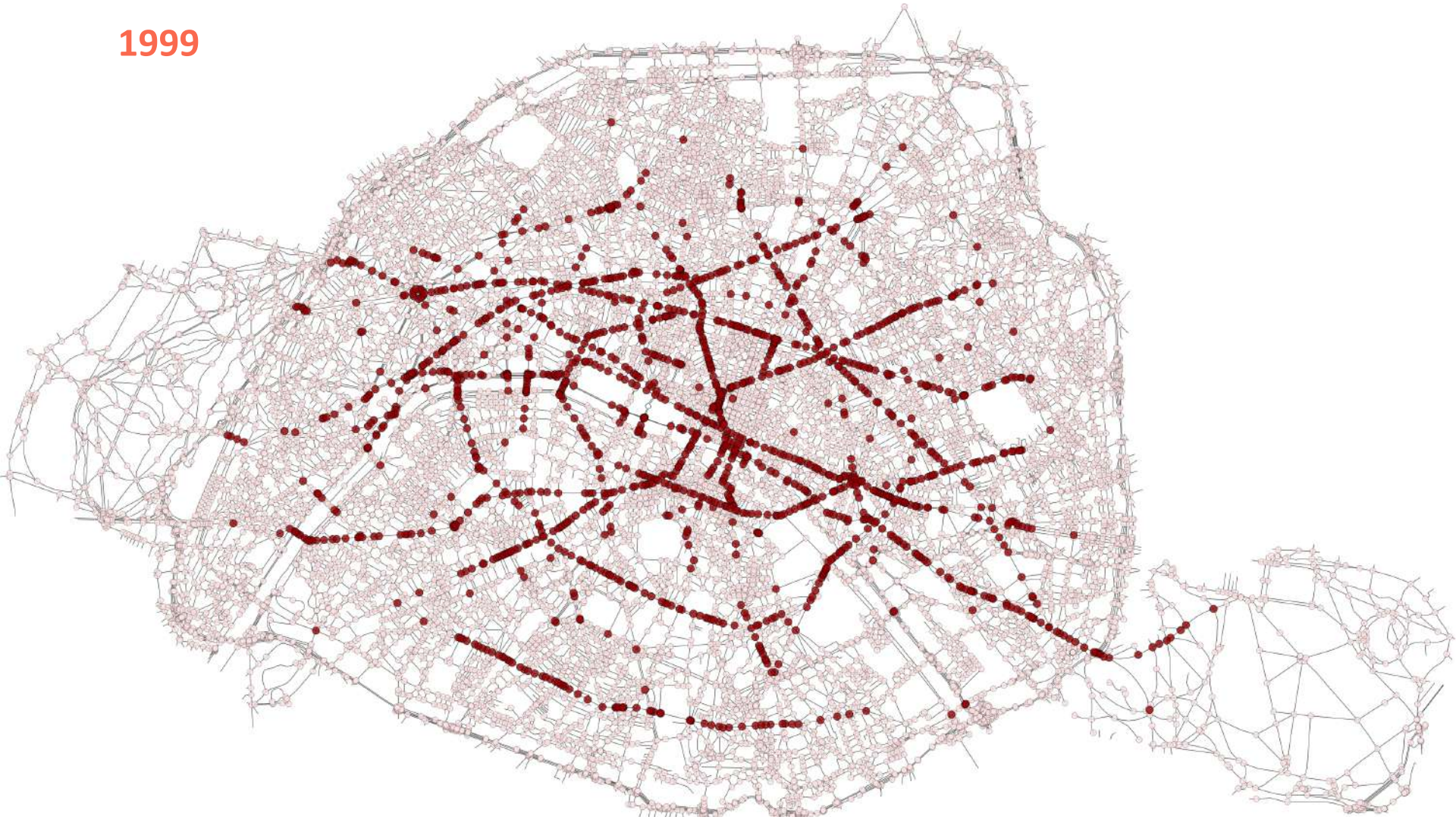
1888 (Haussmann)





# Evolution of Paris

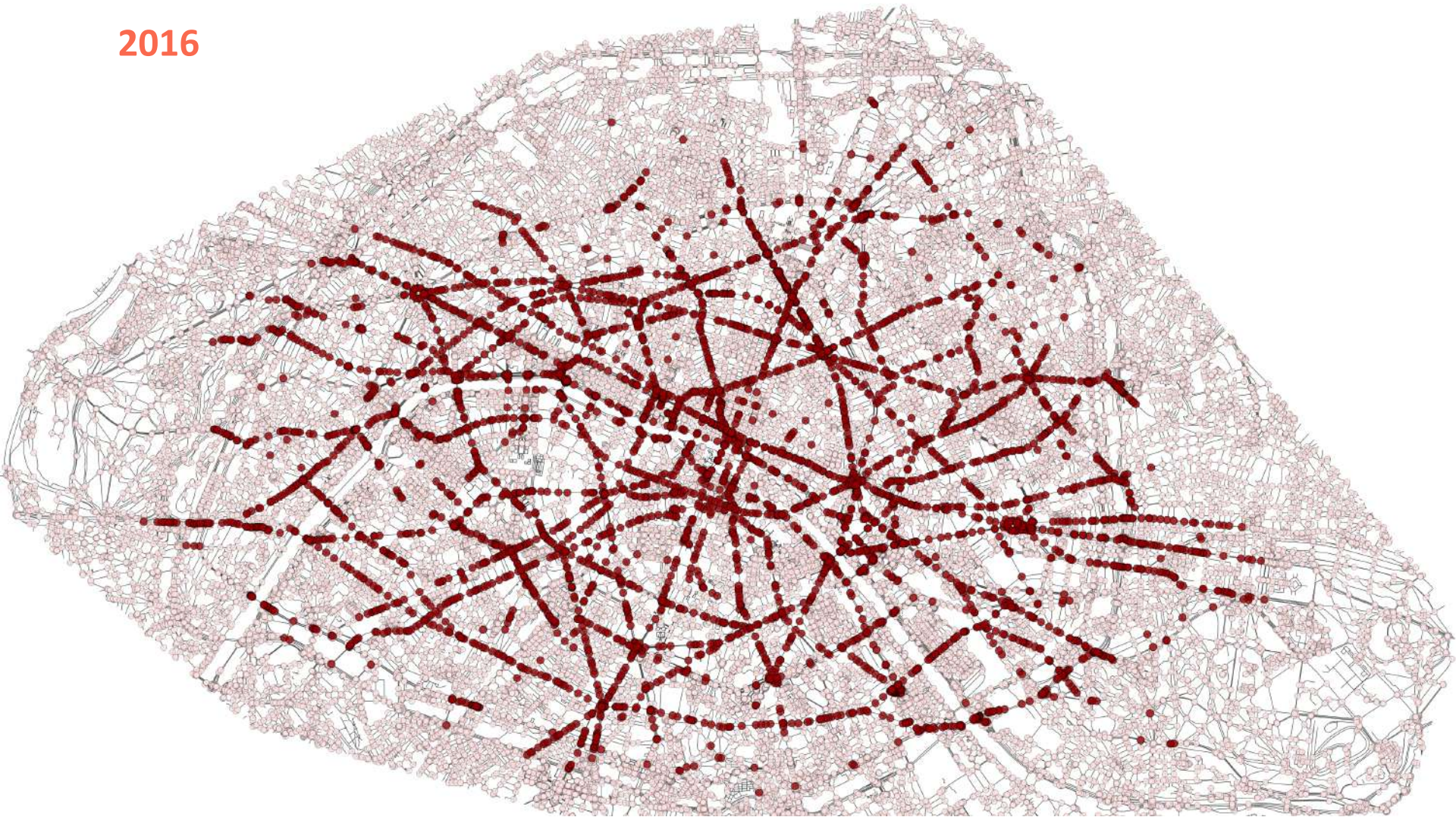
1999





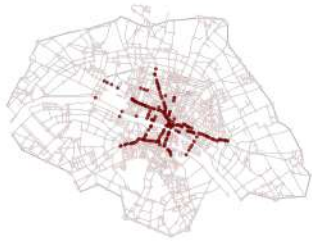
# Evolution of Paris

2016

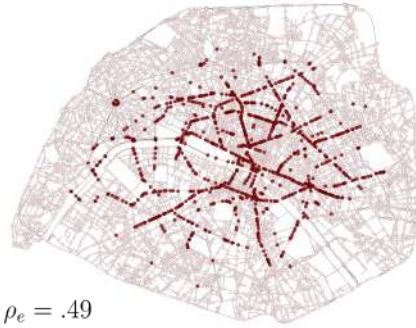




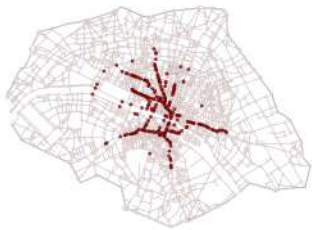
# Evolution of Paris



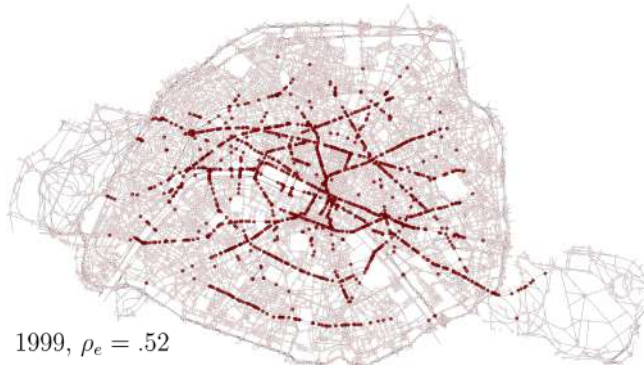
1790,  $\rho_e = .51$



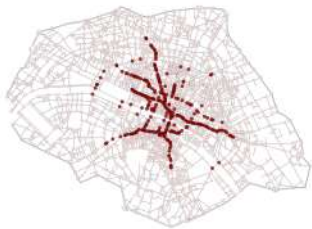
1888,  $\rho_e = .49$



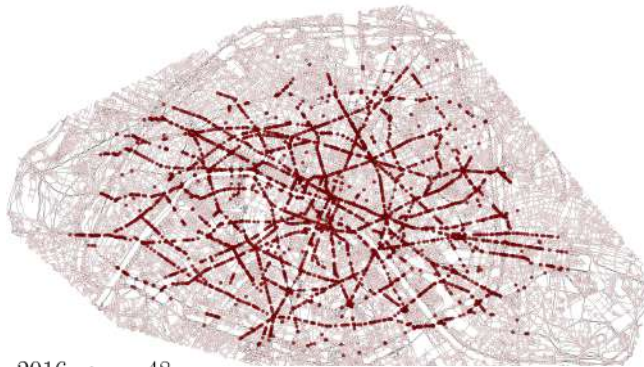
1836,  $\rho_e = .53$



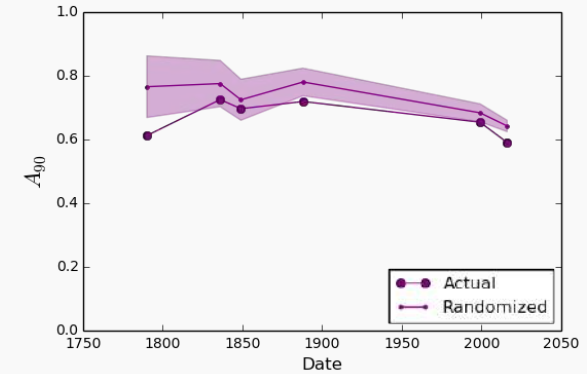
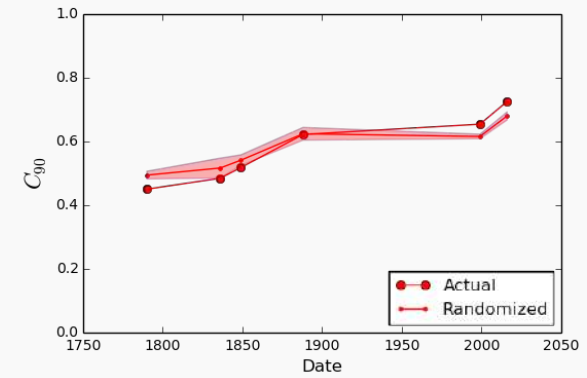
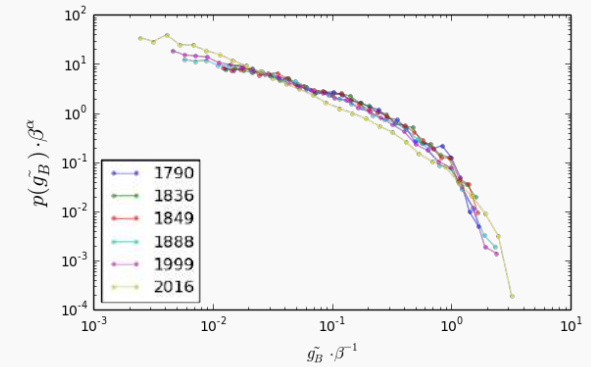
1999,  $\rho_e = .52$



1849,  $\rho_e = .53$



2016,  $\rho_e = .48$



# Takeaways

- Betweenness centralities of streets is sensitive to the scale of measurement.
- At the level of the “full” city, the distribution is bimodal composed of a backbone tree (high betweenness) decorated by loops and dead-ends (low betweenness).
- After a appropriate rescaling, the distribution appears to be invariant across cities, indicating that the “total” flow in cities is a conserved quantity determined entirely by the spatial extent and number of streets.
- On the other hand high betweenness nodes have a complicated spatial dependence, with a “decoupling” between topology and space at a “critical” edge density.
- Results suggest the interesting behavior occurs only in the tail of the distribution. “Neighborhood” of high betweenness nodes of particular interest.
- **Lessons for Central Planners.** Spatial and topological constraints limit room for maneuver. Multimodal transport seems the most efficient choice.

# Human Mobility: Models and Applications

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## Abstract

Recent years have witnessed an explosion of extensive geolocated datasets related to human movement, enabling scientists to quantitatively study individual and collective mobility patterns, as well as generate models that can capture and reproduce the spatiotemporal structures and regularities in human trajectories. The study of human mobility is especially important for applications such as estimating migratory flows, traffic forecasting, urban planning, and epidemic modeling. In this survey, we review some of the historical approaches developed to reproduce various mobility patterns. However, the main focus is on recent works in light of the explosion of relevant data. This review can be used both as an introduction to the fundamental modeling principles of human mobility, and as a collection of technical methods applicable to specific mobility-related problems. The review organizes the subject by differentiating between individual and population mobility and also between short-range and long-range mobility. Throughout the text the description of the theory is intertwined with real-world

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<http://gghoshal.pas.rochester.edu/>